An Overview of Gravitational Self-Force Theory

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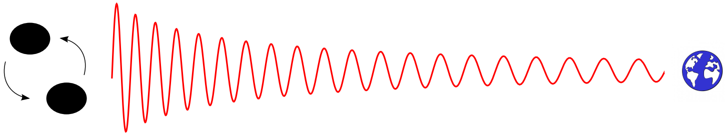


Key takeaways

Contrast with PM/scattering amplitudes:

- Perturbations of a Schwarzschild/Kerr background
  ↳ involved Green's functions, semi-analytics
- Non-perturbative results in the coupling constant G
  ↳ GSF includes exact strong-field information
- No resort to point particles
  ↳ no UV divergence, no regularization

Connection to observables and experiments:

- Gravitational waveforms available (quasicircular inspiral)
- GSF theory applies way beyond the natural regime $\epsilon \ll 1$

Outline

1. Context

2. Foundations

3. State-of-the-art

Outline

The gravitational-wave spectrum

Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

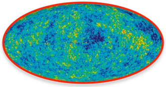
Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

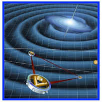
Detectors

Cosmic microwave background polarization



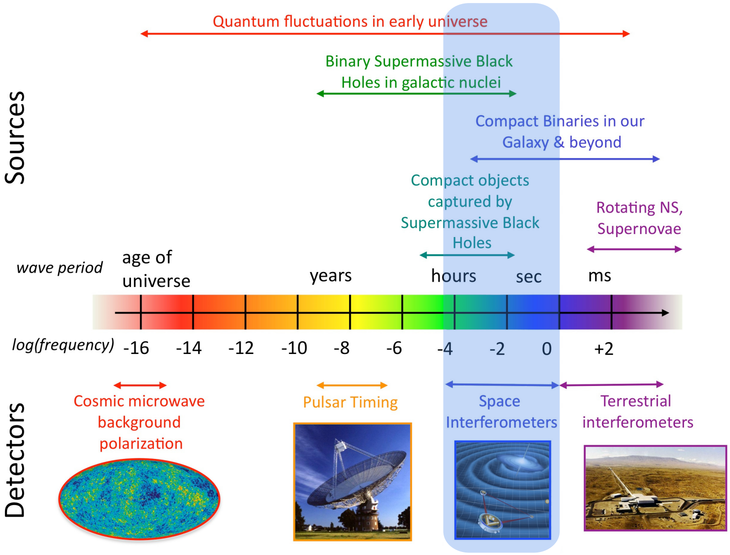
Pulsar Timing

Space Interferometers



Terrestrial interferometers

The gravitational-wave spectrum



Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

wave period age of universe years hours sec ms

log(frequency) -16 -14 -12 -10 -8 -6 -4 -2 0 +2

Detectors

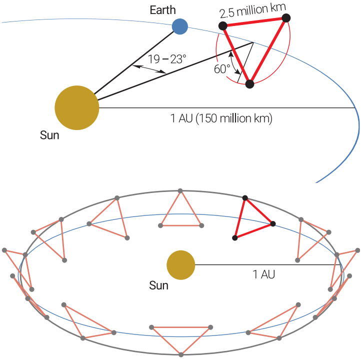
Cosmic microwave background polarization

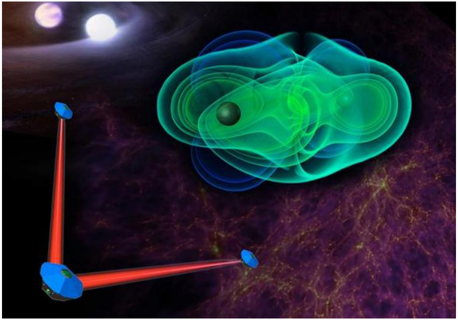
Pulsar Timing

Space Interferometers

Terrestrial interferometers

LISA: a gravitational antenna in space

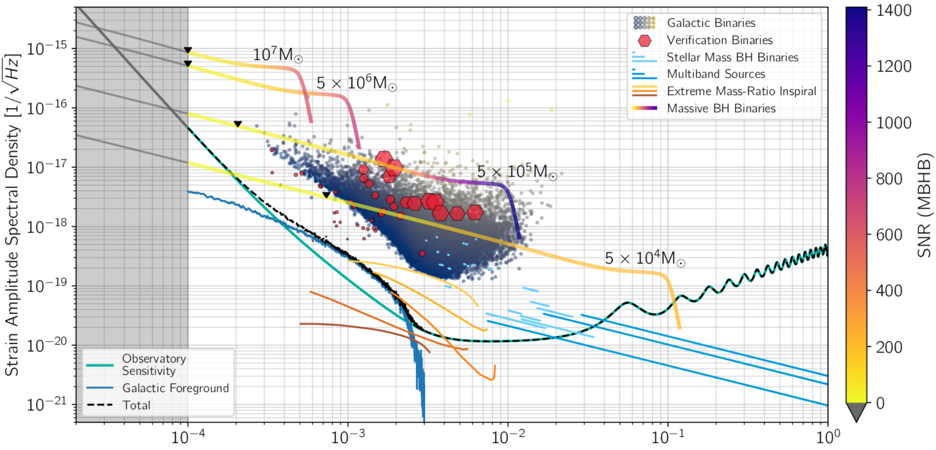




The **LISA mission** was officially adopted/approved by ESA in 2024, with a launch planned for **2035**

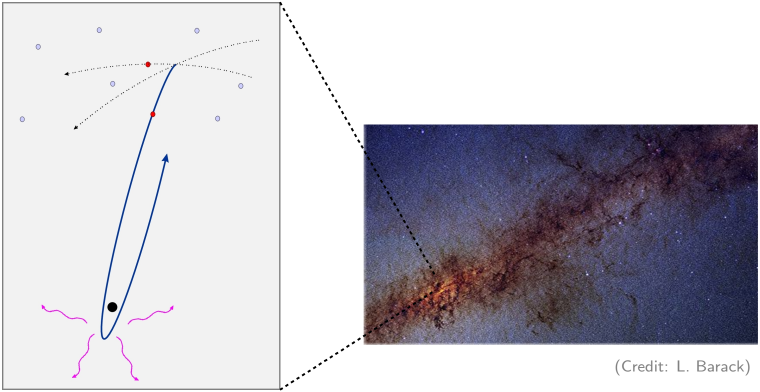
[www.lisamission.org]

LISA sources of gravitational waves



[LISA Collaboration, astro-ph.CO/2402.07571]

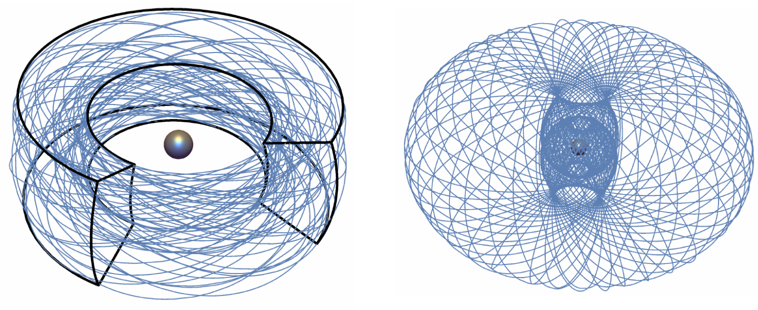
Extreme Mass Ratio Inspirals (EMRIs)



(Credit: L. Barack)

- LISA sensitive to $M \sim 10^5 - 10^7 M_\odot \rightarrow q \equiv M/m \sim 10^3 - 10^6$
- $T_{\text{orb}} \propto M \sim$ hr and $T_{\text{insp}} \propto M^2/m \sim$ yrs $\rightarrow N_{\text{band}} \sim 10^5$ cycles

EMRIs as probes of strong-field geometry

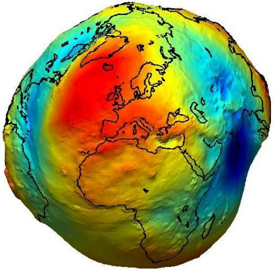


Assuming the central object is a Kerr black hole:

- Orbits are tri-periodic (1 rotation + 2 librations)
- Orbits are ergodic (phase space-filling) in general
- Principal elements undergo conservative secular precession
- Principal elements drift under gravitational radiation-reaction

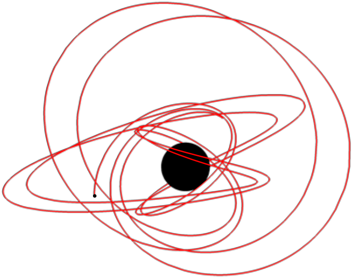
EMRIs as probes of strong-field geometry

Geodesy



$M_{\ell,m}$ arbitrary

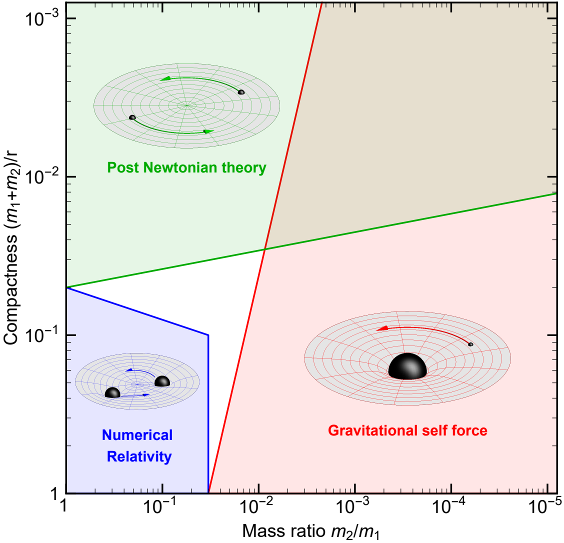
Botromeladesy



$M_{\ell,0} + iS_{\ell,0} = M(ia)^\ell$

Objective: test the black hole **no-hair theorem** of GR

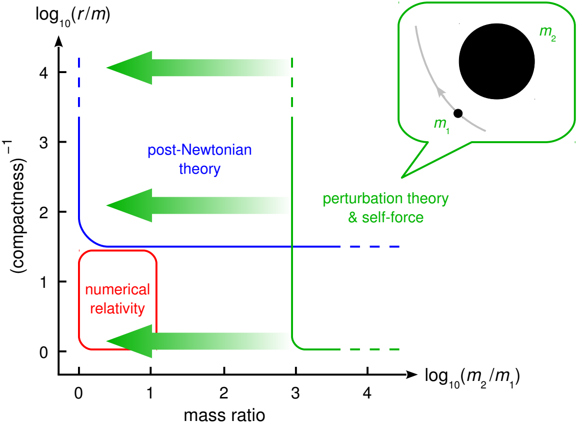
Parameter space of the 2-body problem



[LISA Consortium Waveform Working Group, LRR 2025]

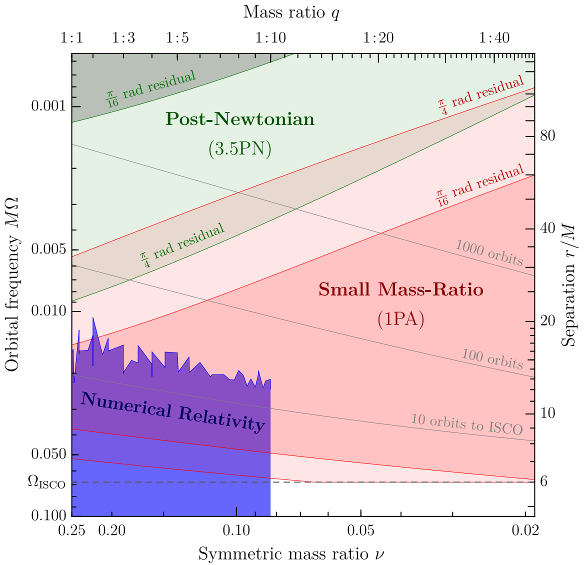
Perturbation theory for comparable masses

Restore **discrete symmetry** by 1 ⇋ 2: $\epsilon \equiv \frac{m_1}{m_2} \quad \rightarrow \quad \nu \equiv \frac{m_1 m_2}{m^2}$



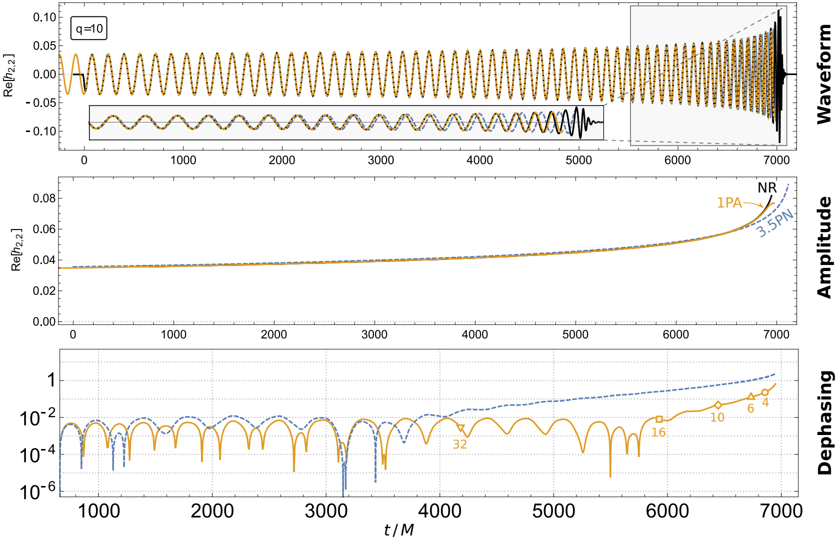
[Le Tiec, IJMPD 2014]

Perturbation theory for comparable masses



[van de Meent & Pfeiffer, PRL 2020]

Perturbation theory for comparable masses



[Wardell, Pound, Warburton, Miller, Durkan & Le Tiec, PRL 2023]

Outline

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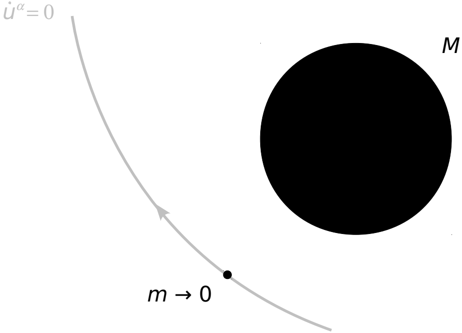
Gravitational self-force: A primer

Spacetime metric

$$g^{\text{exact}}_{\alpha\beta} = g_{\alpha\beta}$$



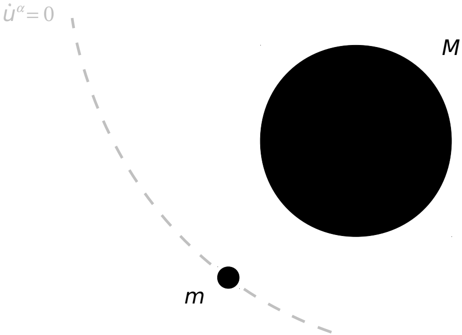
Gravitational self-force: A primer

Spacetime metric

$$g_{\alpha\beta}^{\text{exact}} = g_{\alpha\beta}$$

Small parameter

$$\epsilon \equiv \frac{m}{M} \ll 1$$



Gravitational self-force: A primer

Spacetime metric

$$g^{\text{exact}}_{\alpha\beta} = g_{\alpha\beta} + h_{\alpha\beta}$$

Small parameter

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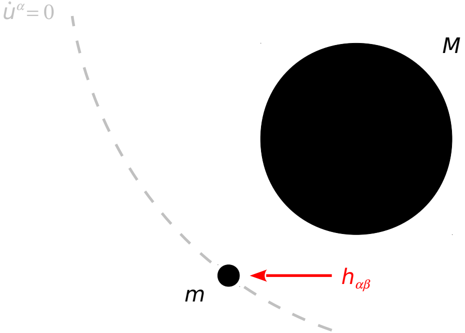

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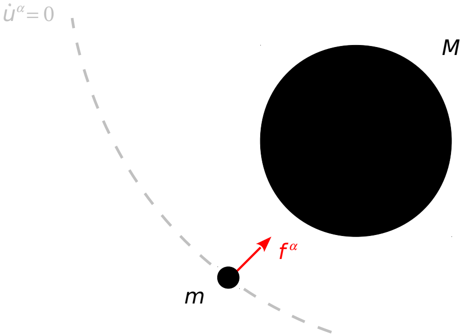

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$$g_{\alpha\beta}^{\text{exact}} = g_{\alpha\beta} + h_{\alpha\beta}$$

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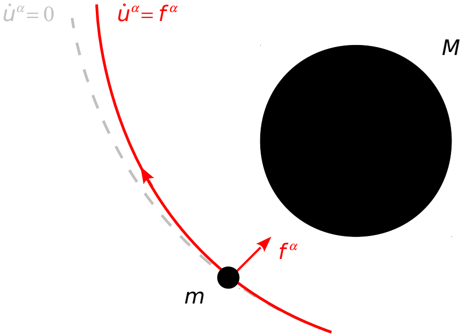
Gravitational self-force

$$\dot{u}^{\alpha} \equiv u^{\beta}\nabla_{\beta}u^{\alpha} = f^{\alpha}[h]$$



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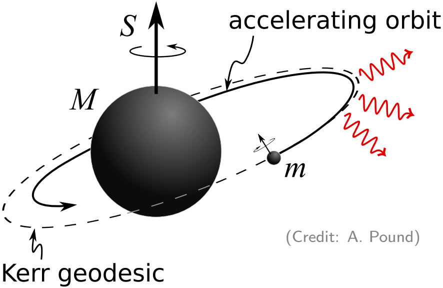
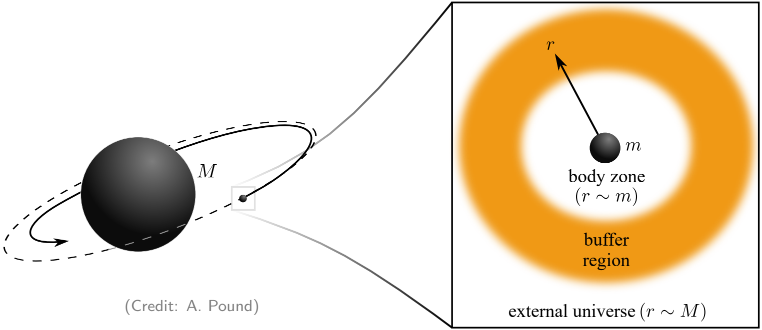


(Credit: A. Pound)

- Dissipative component ⟷ gravitational-wave emission
- Conservative component ⟷ secular precession effects

Matched asymptotic expansions

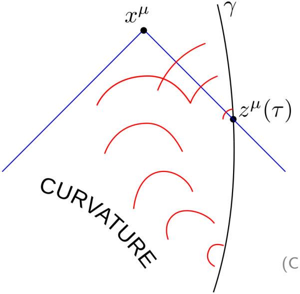


(Credit: A. Pound)

- Trajectory defined on background spacetime using a suitable far-zone limit; constrained by matching near & far expansions
- No resort to point particles: notion *derived* rather than assumed
  ↳ one can associate a worldline γ, four-velocity u^α and mass m

[Mino, Sasaki & Tanaka, PRD 1997 | Poisson, LRR 2004 | Gralla & Wald, PRD 2008]

Post-geodesic equation of motion



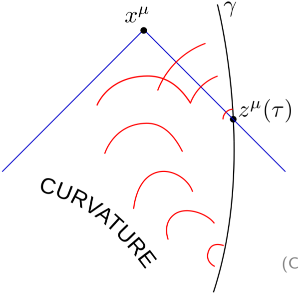
(Credit: A. Pound)

Metric perturbation

$$h_{\alpha\beta} = h_{\alpha\beta}^{\text{direct}} + h_{\alpha\beta}^{\text{tail}}$$

[Mino, Sasaki & Tanaka, PRD 1997 | Quinn & Wald, PRD 1997]

Post-geodesic equation of motion



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Metric perturbation

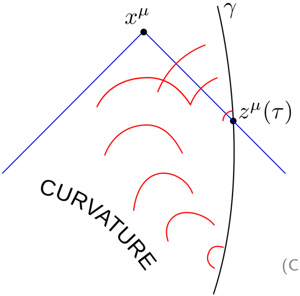
$$h_{\alpha\beta} = h^{\text{direct}}_{\alpha\beta} + h^{\text{tail}}_{\alpha\beta}$$

Tail contribution

$$h^{\text{tail}}_{\alpha\beta}(x) = m\int_{-\infty}^{\tau_-}\left(G_{\alpha\beta\alpha'\beta'} - \tfrac{1}{2}\, g_{\alpha\beta}\, G_{\lambda}{}^{\lambda}{}_{\alpha'\beta'}\right)(x, z(\tau'))\, u^{\alpha'} u^{\beta'}\, \mathrm{d}\tau'$$

[Mino, Sasaki & Tanaka, PRD 1997 | Quinn & Wald, PRD 1997]

Post-geodesic equation of motion



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MiSaTaQuWa equation

$$\dot{u}^{\alpha} = \left(g^{\alpha\beta} + u^{\alpha} u^{\beta}\right) \left(\tfrac{1}{2} \nabla_{\beta} h^{\text{tail}}_{\rho\sigma} - \nabla_{\rho} h^{\text{tail}}_{\beta\sigma}\right)\big|_{z(\tau)}\, u^{\rho} u^{\sigma}$$

[Mino, Sasaki & Tanaka, PRD 1997 | Quinn & Wald, PRD 1997]

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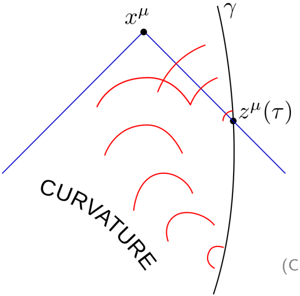

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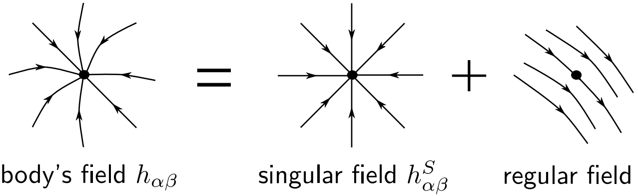
[Mino, Sasaki & Tanaka, PRD 1997 | Quinn & Wald, PRD 1997]

Generalized equivalence principle



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[Detweiler & Whiting, PRD 2003 | Pound, PRD 2012 | Harte, FTP 2015]

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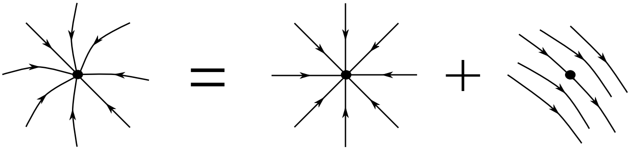


(Credit: A. Pound)

body's field $h_{\alpha\beta}$ singular field $h^S_{\alpha\beta}$ regular field $h^R_{\alpha\beta}$

singular/self field

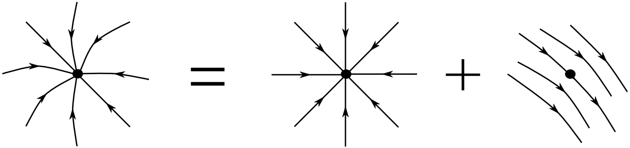
$$h^S \sim m/r$$

$$\Box h^S \sim -16\pi T$$

$$f^\alpha[h^S] = 0$$

[Detweiler & Whiting, PRD 2003 | Pound, PRD 2012 | Harte, FTP 2015]

Generalized equivalence principle



(Credit: A. Pound)

body's field $h_{\alpha\beta}$ = singular field $h^S_{\alpha\beta}$ + regular field $h^R_{\alpha\beta}$

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regular/residual field

$$h^R \sim h^{\text{tail}} + \text{local terms}$$

$$\Box h^R \sim 0$$

$$\dot{u}^\alpha = f^\alpha[h^R]$$

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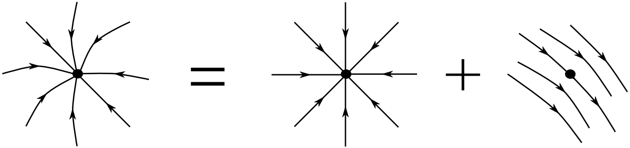
Generalized equivalence principle



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self-accelerated motion in $g_{\alpha\beta}$ $\Longleftrightarrow$ **geodesic motion** in $g_{\alpha\beta} + h^R_{\alpha\beta}$

[Detweiler & Whiting, PRD 2003 | Pound, PRD 2012 | Harte, FTP 2015]

Einstein field equations

- Expand the exact metric about a given Kerr background $g_{\alpha\beta}$:

$$g_{\alpha\beta}^{\text{exact}} = g_{\alpha\beta} + \epsilon\, h_{\alpha\beta}^{(1)} + \epsilon^2\, h_{\alpha\beta}^{(2)} + O(\epsilon^3)$$

Einstein field equations

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$$g^{\text{exact}}_{\alpha\beta} = g_{\alpha\beta} + \epsilon\, h^{(1)}_{\alpha\beta} + \epsilon^2\, h^{(2)}_{\alpha\beta} + O(\epsilon^3)$$

- Expand the Einstein tensor $G_{\alpha\beta}$ and the energy-momentum tensor $T_{\alpha\beta}$ accordingly, and equate order by order in ϵ:

$$\begin{aligned}
O(\epsilon^0):& \qquad & G_{\alpha\beta}[g] &= 0 \\
O(\epsilon^1):& \qquad & \delta G_{\alpha\beta}[h^{(1)}] &= 8\pi\, T^{(1)}_{\alpha\beta} \\
O(\epsilon^2):& \qquad & \underbrace{\delta G_{\alpha\beta}[h^{(2)}]}_{\propto \Box h^{(2)}_{\alpha\beta} + \cdots} &= 8\pi\, T^{(2)}_{\alpha\beta} - \underbrace{\delta^2 G_{\alpha\beta}[h^{(1)}, h^{(1)}]}_{\sim m^2/r^4}
\end{aligned}$$

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\end{aligned}$$

- Failure of the point-particle description at second order in ϵ
  ↳ decompose into puncture/residual fields: *effective source*

Two-timescale expansion

- Separation of timescales during inspiral: $T_{\rm orb}/T_{\rm r.r.} \sim \epsilon \ll 1$

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

Two-timescale expansion

- Separation of timescales during inspiral: $T_{\mathrm{orb}}/T_{\mathrm{r.r.}} \sim \epsilon \ll 1$
- Introduce a fast time t and a slow time $\tilde{t} \equiv \epsilon t$, treated as *independent* variables

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

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- The angles/phases $\boldsymbol{\psi}(t)$ are fast variables and the actions $\mathbf{J}(\tilde{t})$ are slow variables

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

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- The angles/phases $\psi(t)$ are fast variables and the actions $\mathbf{J}(\tilde{t})$ are slow variables
- The two-timescale expansion of the metric then reads

$$g^{\rm exact}_{\alpha\beta}(t,\mathbf{x},\epsilon) = g_{\alpha\beta}(\mathbf{x}) + \epsilon h^{(1)}_{\alpha\beta}(\mathbf{x},\mathbf{J},\psi) + \epsilon^2 h^{(2)}_{\alpha\beta}(\mathbf{x},\mathbf{J},\psi) + O(\epsilon^3)$$

$$\text{where} \quad h^{(n)}_{\alpha\beta} = \sum_{\mathbf{k}} \underbrace{h^{(n,\mathbf{k})}_{\alpha\beta}(\mathbf{x},\mathbf{J})}_{\substack{\text{slowly evolving}\\ \text{amplitude}}} \times \underbrace{e^{-i\mathbf{k}\cdot\psi}}_{\substack{\text{rapidly evolving}\\ \text{phase}}}$$

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

Post-adiabatic orbit evolution

- The actions $\mathbf{J}$ and angles/phases $\boldsymbol{\psi}$ evolve according to

$$\frac{\mathrm{d}\mathbf{J}}{\mathrm{d}t} = \sum_{\mathbf{k}} \left[\epsilon\, \mathbf{G}_{(1,\mathbf{k})}(\tilde{t}) + \epsilon^2\, \mathbf{G}_{(2,\mathbf{k})}(\tilde{t}) + O(\epsilon^3)\right] e^{-i\mathbf{k}\cdot\boldsymbol{\psi}}$$

$$\frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}t} = \boldsymbol{\Omega}(\mathbf{J}) = \boldsymbol{\Omega}_{(0)}(\tilde{t}) + \epsilon\,\boldsymbol{\Omega}_{(1)}(\tilde{t}) + O(\epsilon^2)$$

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

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- This implies the *post-adiabatic* phase evolution

$$\boldsymbol{\psi}(t) = \frac{1}{\epsilon}\left[\boldsymbol{\psi}_{(0)}(\tilde{t}) + \epsilon\, \boldsymbol{\psi}_{(1)}(\tilde{t}) + O(\epsilon^2)\right]$$

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

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Adiabatic (0PA)

- averaged first-order dissipative self-force

Post-Adiabatic (1PA)

- oscillatory first-order dissipative
- complete first-order conservative
- averaged second-order dissipative

[Hinderer & Flanagan, PRD 2008 | Miller & Pound, PRD 2021]

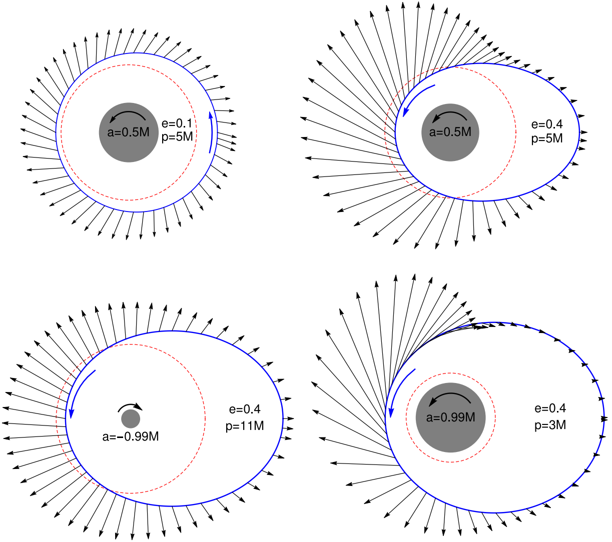
Other technical aspects not covered

- Use of near-identity transformation to remove fast oscillations
- Slow evolution of the BH mass $M(\tilde{t})$ and spin $S(\tilde{t})$ at $O(\epsilon^2)$
- Need to match to a post-Minkowskian expansion near $\mathcal{I}^+$
- Need to match to a near-horizon expansion near $\mathcal{H}^+$
- Transition to final plunge when two-timescale breaks down
- Transitions through orbital resonances for a Kerr background
- . . .

Outline

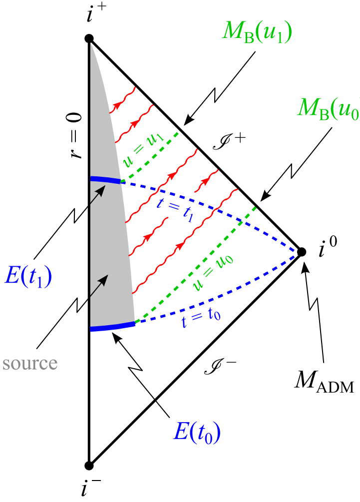
Self-force along fixed geodesic orbits

Sample results for equatorial orbits in Kerr



[van de Meent, PRD 2016]

Orbital evolution *via* energy balance



- Bondi-Sachs mass-loss formula

$$\frac{\mathrm{d}M_B}{\mathrm{d}u} = -\mathcal{F}(u)$$

- Gravitational binding energy

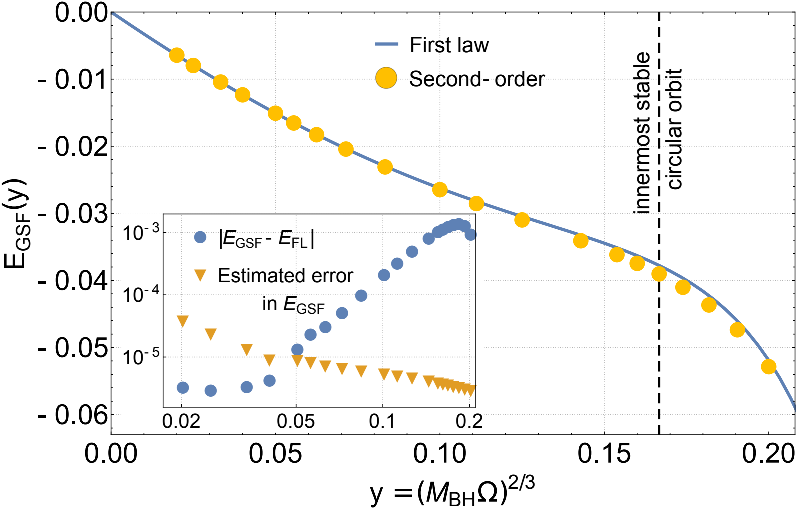
$$E \equiv M_B - M - m$$

- Orbital frequency evolution

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = -\frac{\mathcal{F}(\Omega)}{E'(\Omega)} + \cdots$$

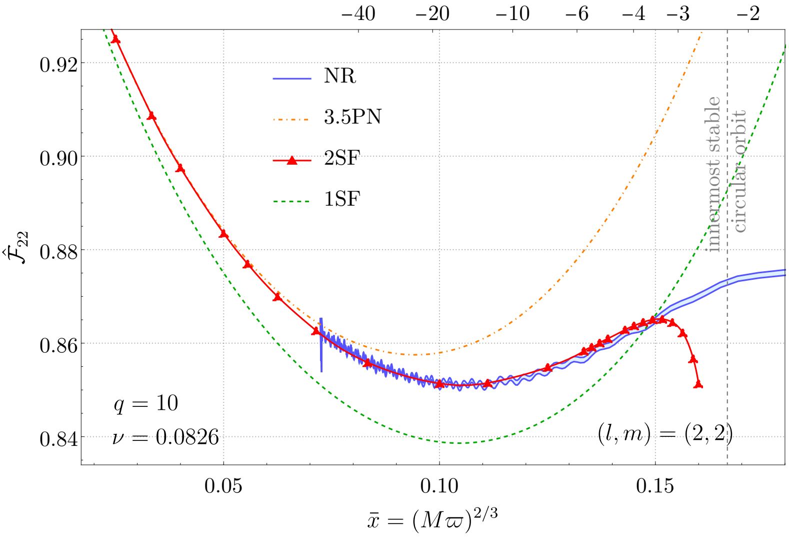
[Le Tiec, Blanchet & Whiting, PRD 2012]

Binding energy *vs* orbital frequency



[Le Tiec, Buonanno & Barausse, PRL 2012 | Pound, Wardell, Warburton & Miller, PRL 2019]

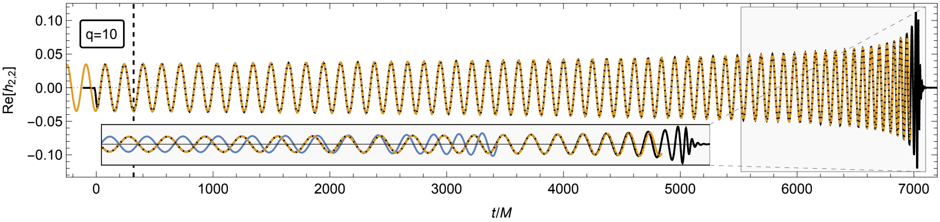
GW energy flux *vs* orbital frequency

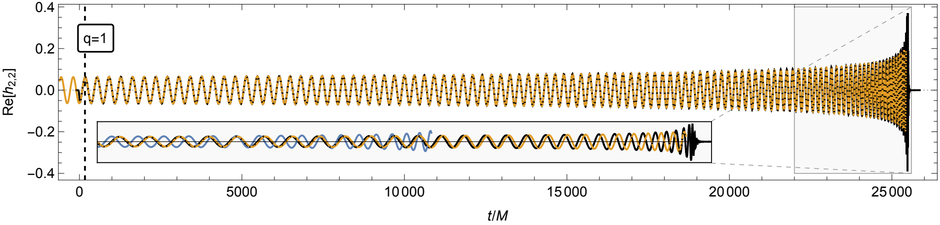


[Warburton, Pound, Wardell, Miller & Durkan, PRL 2021]

Post-adiabatic gravitational waveforms

Nonspinning binaries on quasicircular orbits

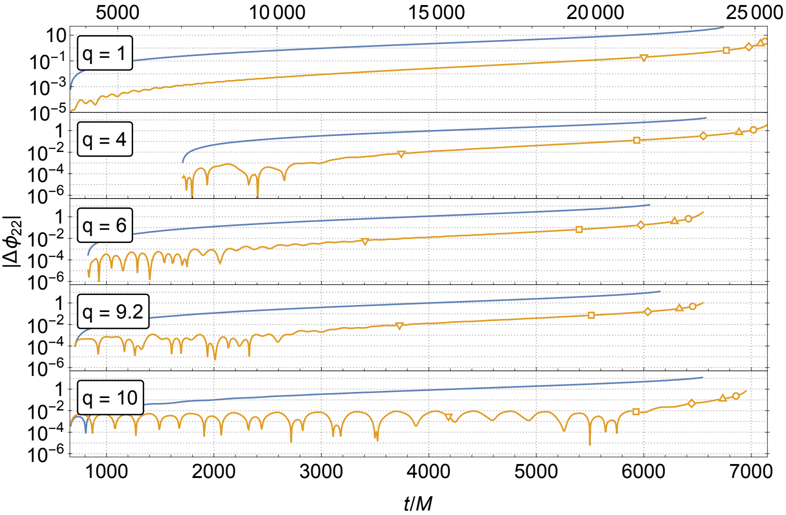




[Wardell, Pound, Warburton, Miller, Durkan & Le Tiec, PRL 2023]

Post-adiabatic gravitational waveforms

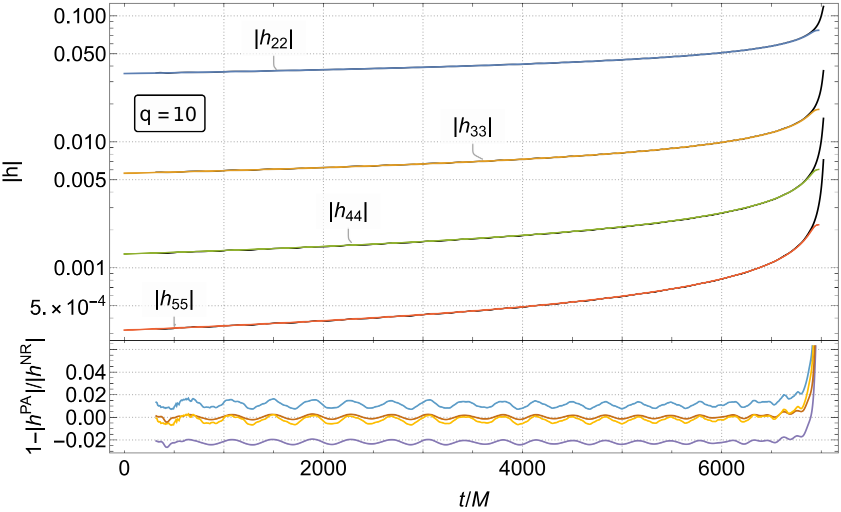
Accumulated dephasing



[Wardell, Pound, Warburton, Miller, Durkan & Le Tiec, PRL 2023]

Post-adiabatic gravitational waveforms

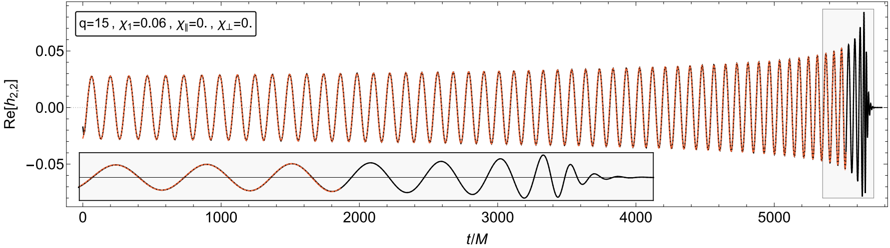
Mode waveform amplitudes

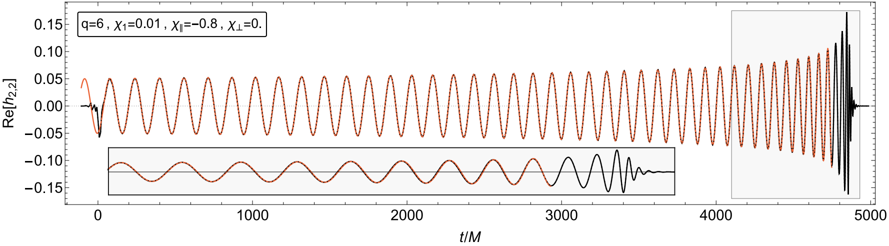


[Wardell, Pound, Warburton, Miller, Durkan & Le Tiec, PRL 2023]

Post-adiabatic gravitational waveforms

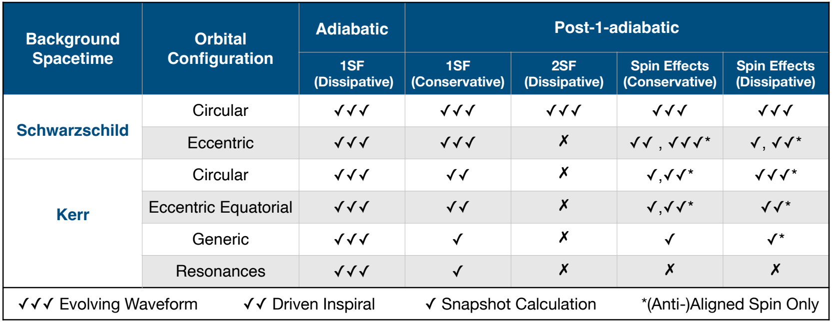
Nonprecessing, spinning binaries on quasicircular orbits





[Mathews, Wardell, Pound & Warburton, gr-qc/2510.16113]

Progress and challenges



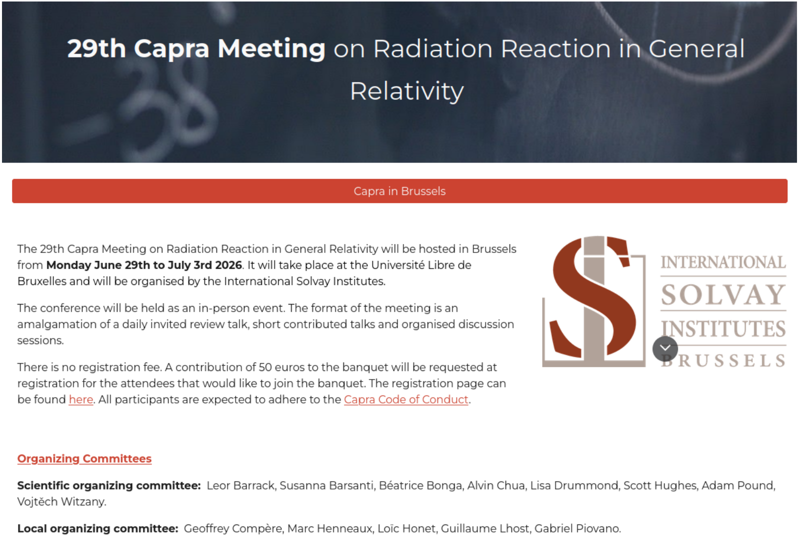
Background Spacetime	Orbital Configuration	Adiabatic	Post-1-adiabatic			
		1SF (Dissipative)	1SF (Conservative)	2SF (Dissipative)	Spin Effects (Conservative)	Spin Effects (Dissipative)
Schwarzschild	Circular	✓✓✓	✓✓✓	✓✓✓	✓✓✓	✓✓✓
	Eccentric	✓✓✓	✓✓✓	✗	✓✓ , ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	✗	✓,✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✓✓	✗	✓,✓✓*	✓✓*
	Generic	✓✓✓	✓	✗	✓	✓*
	Resonances	✓✓✓	✓	✗	✗	✗

✓✓✓ Evolving Waveform ✓✓ Driven Inspiral ✓ Snapshot Calculation *(Anti-)Aligned Spin Only

- Generic bound orbits in Kerr
- Transition accross resonances
- Inclusion of the final plunge
- Scattering orbits
- Environmental effects
- Beyond general relativity

[LISA Consortium Waveform Working Group, LRR 2025]

Capra Meetings on Radiation Reaction



29th Capra Meeting on Radiation Reaction in General Relativity

Capra in Brussels

The 29th Capra Meeting on Radiation Reaction in General Relativity will be hosted in Brussels from **Monday June 29th to July 3rd 2026**. It will take place at the Université Libre de Bruxelles and will be organised by the International Solvay Institutes.

The conference will be held as an in-person event. The format of the meeting is an amalgamation of a daily invited review talk, short contributed talks and organised discussion sessions.

There is no registration fee. A contribution of 50 euros to the banquet will be requested at registration for the attendees that would like to join the banquet. The registration page can be found here. All participants are expected to adhere to the Capra Code of Conduct.

Organizing Committees

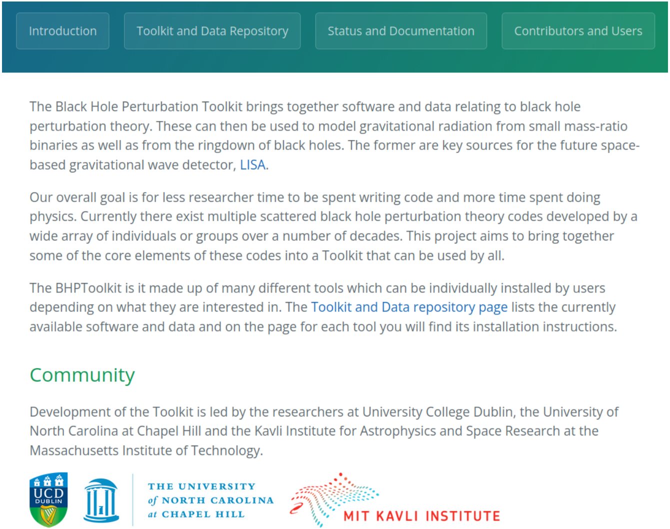
Scientific organizing committee: Leor Barrack, Susanna Barsanti, Béatrice Bonga, Alvin Chua, Lisa Drummond, Scott Hughes, Adam Pound, Vojtěch Witzany.

Local organizing committee: Geoffrey Compère, Marc Henneaux, Loïc Honet, Guillaume Lhost, Gabriel Piovano.

[www.caprameeting.org]

Black Hole Perturbation Toolkit

Open tools for black hole perturbation theory



The Black Hole Perturbation Toolkit brings together software and data relating to black hole perturbation theory. These can then be used to model gravitational radiation from small mass-ratio binaries as well as from the ringdown of black holes. The former are key sources for the future space-based gravitational wave detector, LISA.

Our overall goal is for less researcher time to be spent writing code and more time spent doing physics. Currently there exist multiple scattered black hole perturbation theory codes developed by a wide array of individuals or groups over a number of decades. This project aims to bring together some of the core elements of these codes into a Toolkit that can be used by all.

The BHPToolkit is it made up of many different tools which can be individually installed by users depending on what they are interested in. The Toolkit and Data repository page lists the currently available software and data and on the page for each tool you will find its installation instructions.

Community

Development of the Toolkit is led by the researchers at University College Dublin, the University of North Carolina at Chapel Hill and the Kavli Institute for Astrophysics and Space Research at the Massachusetts Institute of Technology.

[bhptoolkit.org]

Some review articles

- Leor Barack
  Gravitational self-force in extreme mass-ratio inspirals
  Class. Quantum Grav. **26**, 213001 (2009)
- Eric Poisson, Adam Pound & Ian Vega
  The motion of point particles in curved spacetimes
  Living Rev. Relativ. **14**, 7 (2011)
- Abraham I. Harte
  Motion in classical field theories and the foundations of the SF problem
  Fund. Theor. Phys. **179**, 327 (2015)
- Leor Barack & Adam Pound
  Self-force and radiation reaction in general relativity
  Rep. Prog. Phys. **82**, 016904 (2019)
- Adam Pound & Barry Wardell
  Black hole perturbation theory and gravitational self-force
  In: Handbook of Gravitational Wave Astronomy, Springer (2021)
- LISA Consortium Waveform Working Group
  Waveform modelling for the Laser Interferometer Space Antenna
  Living Rev. Relativ. **28**, 9 (2025)

Additional Material

Comparisons to numerical relativity

Relativistic orbital dynamics

- Periastron advance [Le Tiec *et al.*, PRL 2011; PRD 2013]
- Binding energy [Le Tiec, Buonanno & Barausse, PRL 2012]
- Surface gravity [Zimmerman, Lewis & Pfeiffer, PRL 2016]
  [Le Tiec & Grandclément, CQG 2018]

Gravitational-wave emission

- Recoil velocity [Nagar, PRD 2013]
- Head-on waveform [Sperhake *et al.*, PRD 2011]
- Inspiral energy flux [Warburton *et al.*, PRL 2021]
- Inspiral waveform [Ramos-Buades *et al.*, PRD 2022]
  [Islam & Khanna, PRD 2023]

Why does BHPT perform so well?

- In perturbation theory, one traditionally expands as

$$f(\Omega; m_i) = \sum_{k=0}^{k_{\text{max}}} a_k(m_2\Omega)\,\epsilon^k \quad \text{where} \quad \epsilon \equiv m_1/m_2 \in [0,1]$$

- However, most physically interesting relationships $f(\Omega; m_i)$ are symmetric under exchange $m_1 \leftrightharpoons m_2$
- Hence, a better-motivated expansion is

$$f(\Omega; m_i) = \sum_{k=0}^{k_{\text{max}}} b_k(m\Omega)\,\nu^k \quad \text{where} \quad \nu \equiv m_1 m_2/m^2 \in [0,1/4]$$

- In a PN expansion, we have $b_n = \mathcal{O}\big(1/c^{2n}\big) = n\text{PN} + \cdots$

[Le Tiec, IJMPD 2014]