# Gravitational waves, relativistic celestial mechanics and black hole physics

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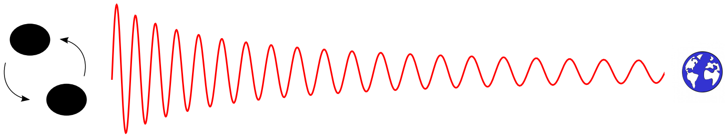
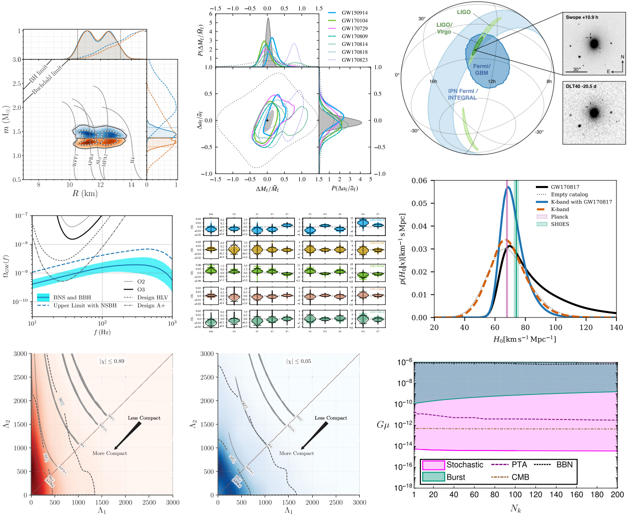


# The beginnings of gravitational-wave science



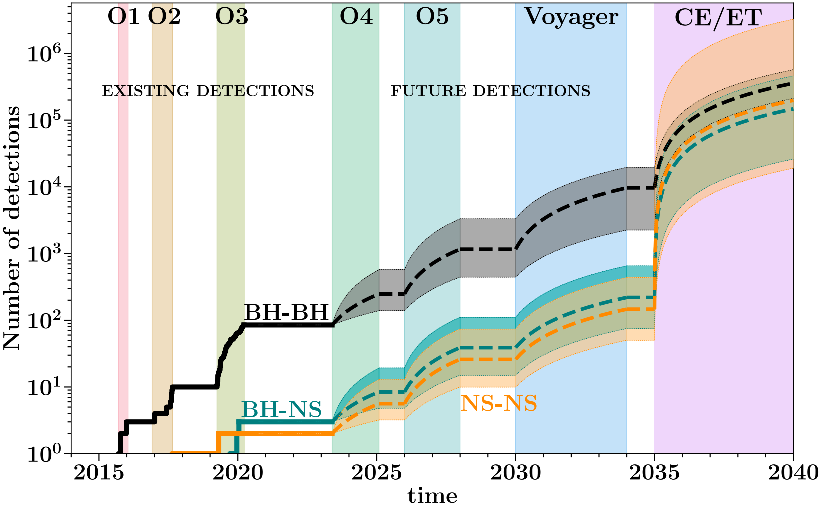




**Science with gravitational-wave observations**

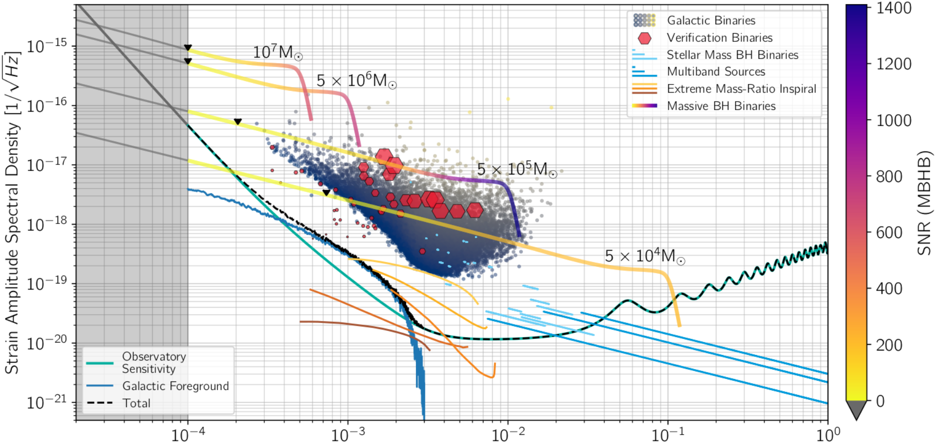
**Detectors**

# The future of gravitational-wave science



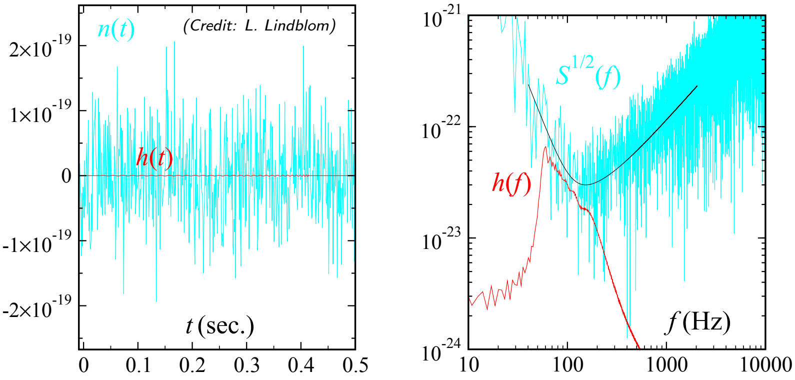

[Broekgaarden, astro-ph.HE/2303.17628]

# The future of gravitational-wave science



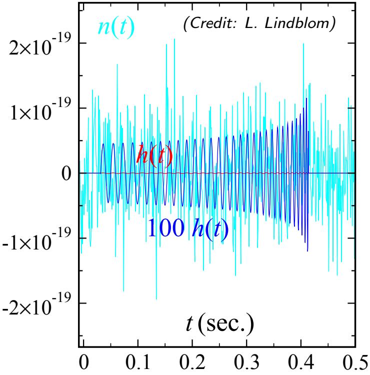

[LISA Collaboration, astro-ph.CO/2402.07571]

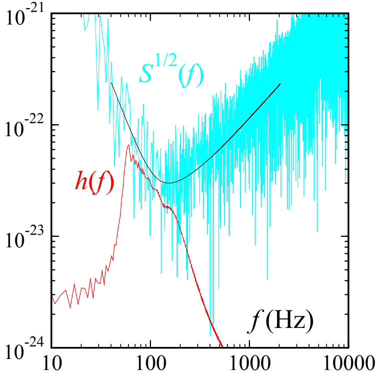
# Need for highly accurate template waveforms




If the expected signal is *known in advance* then $n(t)$ can be filtered and $h(t)$ recovered by matched filtering $\longrightarrow$ **template waveforms**

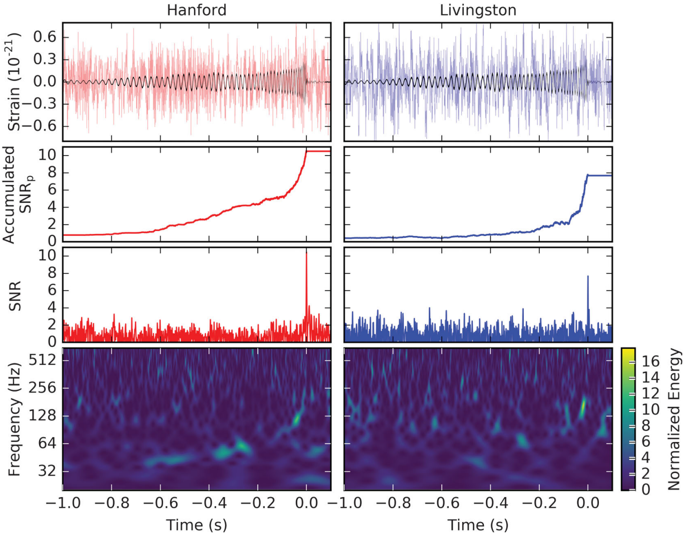
# Need for highly accurate template waveforms




If the expected signal is *known in advance* then $n(t)$ can be filtered and $h(t)$ recovered by matched filtering $\longrightarrow$ **template waveforms**

# An example: the event GW151226



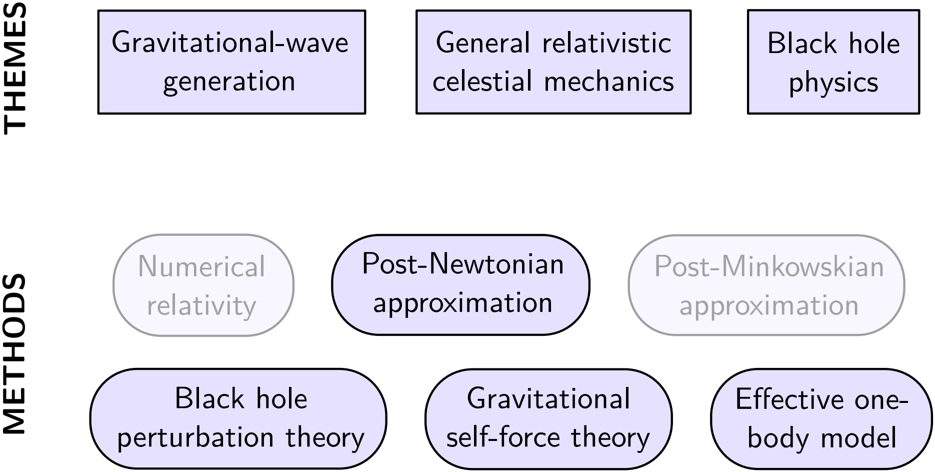

[LIGO-Virgo Collaboration, PRL 2016]

**THEMES**

Gravitational-wave generation

General relativistic celestial mechanics

Black hole physics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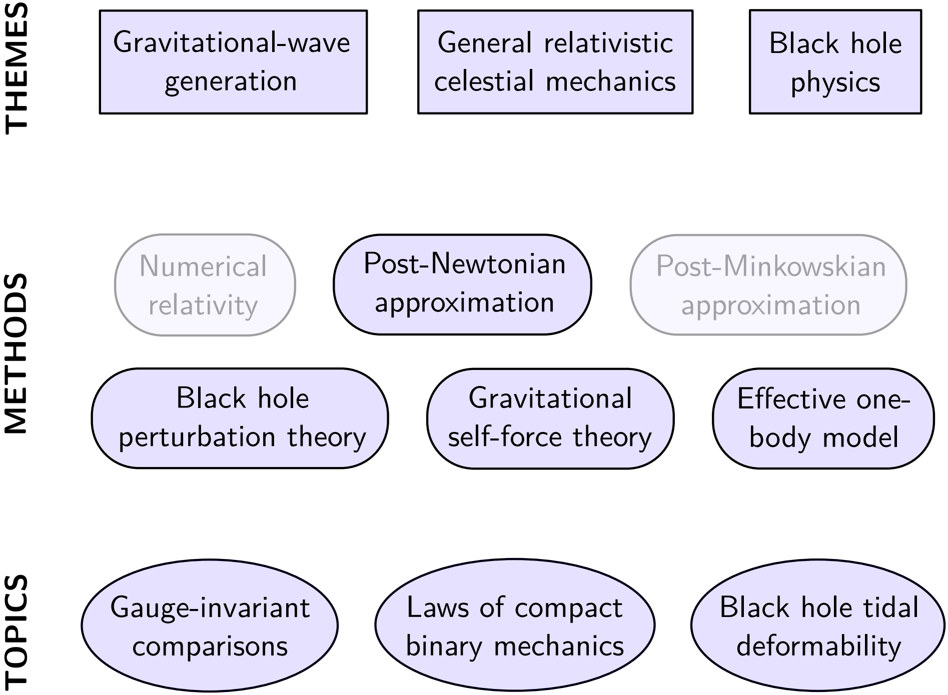
Numerical relativity
Post-Newtonian approximation
Post-Minkowskian approximation
Black hole perturbation theory
Gravitational self-force theory
Effective one-body model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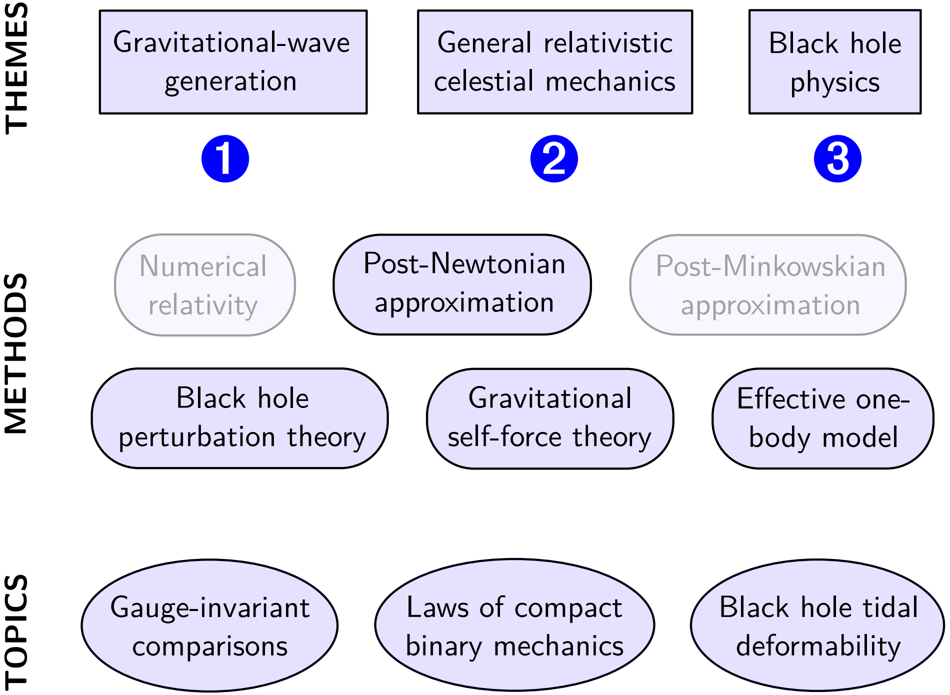
Gauge-invariant comparisons
Laws of compact binary mechanics
Black hole tidal deformability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# Outline

1. Universal class of template waveforms
2. First law of compact binary mechanics
3. The shape of interacting black holes

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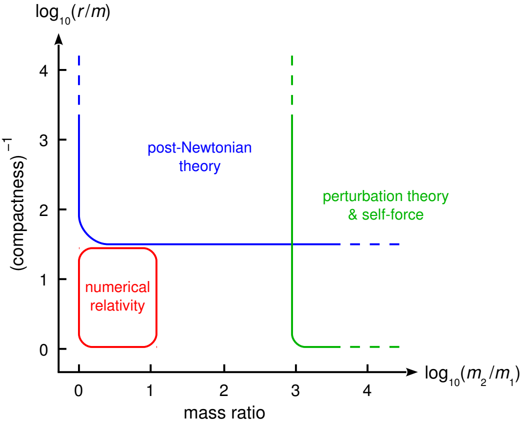
# Modeling coalescing compact binaries




[LISA Waveform Working Group, gr-qc/2311.0130]

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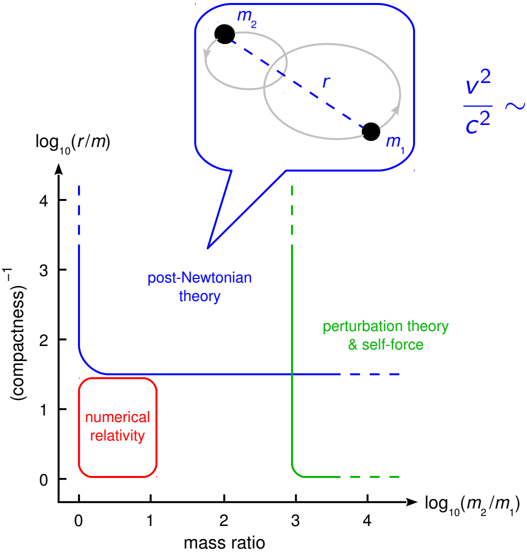
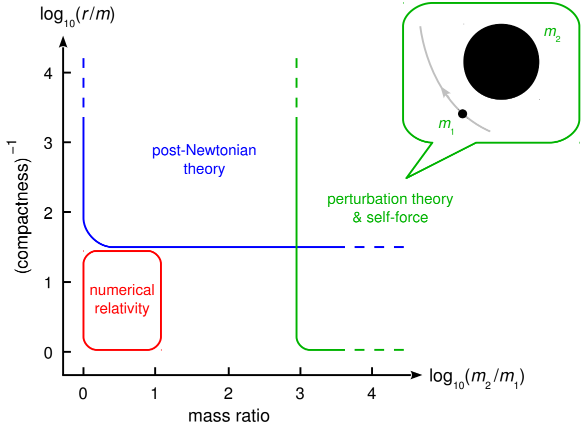



$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \ll 1$$

[LISA Waveform Working Group, gr-qc/2311.0130]

# Modeling coalescing compact binaries

$$q \equiv \frac{m_1}{m_2} \ll 1$$

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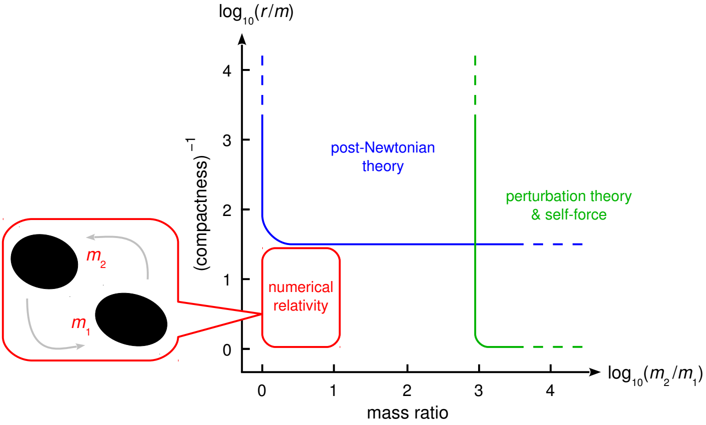


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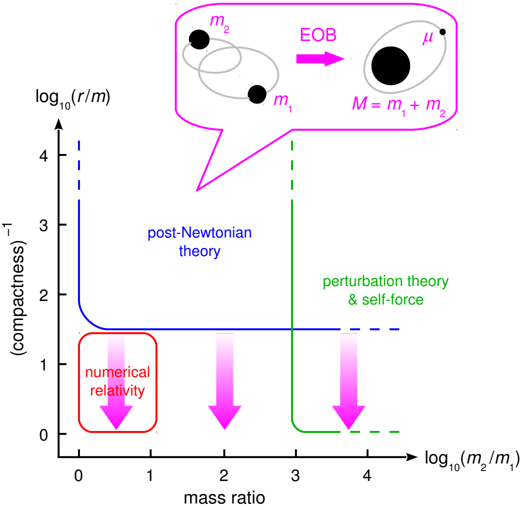
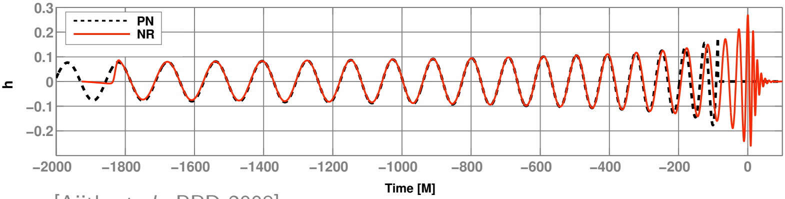



[LISA Waveform Working Group, gr-qc/2311.0130]

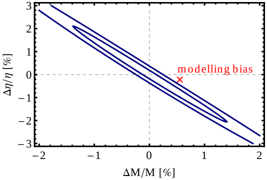
# Main shortcomings of current waveforms




[Ajith *et al.*, PRD 2008]

*For 3G detectors we find that the mismatch error for semi-analytical models needs to be reduced by at least three orders of magnitude and for NR waveforms by one order of magnitude.*

[Pürrer & Haster, PRR 2020]




[Ohme, CQG 2012]

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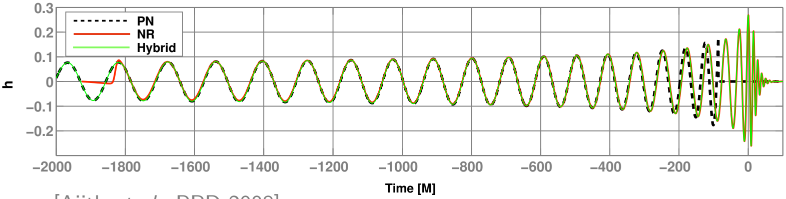


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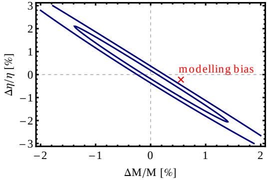



[Ohme, CQG 2012]

# Systematic uncertainties in modeling IMRIs

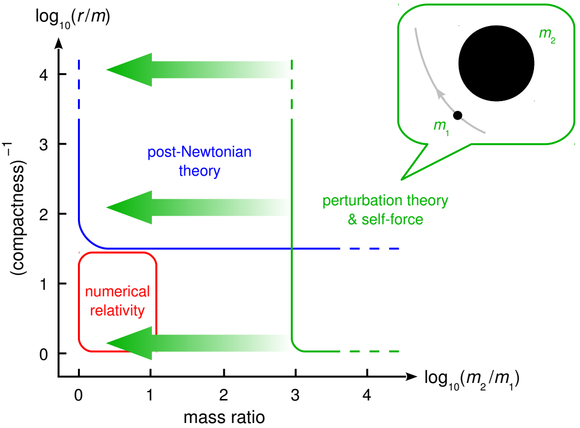
*The mass ratio of GW191219_163120's source is inferred to be $q = 0.038^{+0.005}_{-0.004}$, which is extremely challenging for waveform modeling, and thus there may be systematic uncertainties in results for this candidate.*

*Modeling of higher-order multipole moments is particularly important for inferring the properties of systems with unequal masses, and may impact inference of parameters including the mass ratio, inclination and distance.*

[LIGO-Virgo-KAGRA Collaborations, PRX 2023]

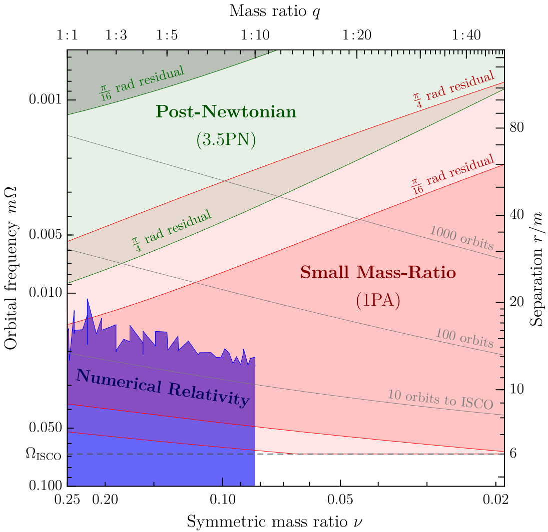
# Perturbation theory for comparable masses

Restore **discrete symmetry** by 1 ⇋ 2: $q \equiv \frac{m_1}{m_2} \rightarrow \nu \equiv \frac{m_1 m_2}{m^2}$




[Le Tiec, IJMPD 2014]

# Perturbation theory for comparable masses




[van de Meent & Pfeiffer, PRL 2020]

# Comparisons to numerical relativity

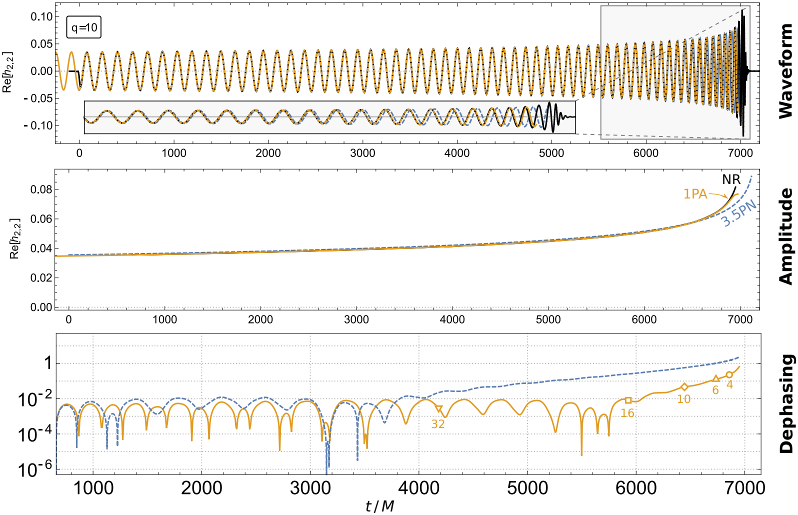
## Relativistic orbital dynamics

- Periastron advance [Le Tiec *et al.*, PRL 2011; PRD 2013]
- Binding energy [Le Tiec, Buonanno & Barausse, PRL 2012]
- Surface gravity [Zimmerman, Lewis & Pfeiffer, PRL 2016]
  [Le Tiec & Grandclément, CQG 2018]

## Gravitational-wave emission

- Recoil velocity [Nagar, PRD 2013]
- Head-on waveform [Sperhake *et al.*, PRD 2011]
- Inspiral energy flux [Warburton *et al.*, PRL 2021]
- Inspiral waveform [Ramos-Buades *et al.*, PRD 2022]
  [Islam & Khanna, PRD 2023]

# Post-adiabatic gravitational waveforms




[Wardell, Pound, Warburton, Miller, Durkan & Le Tiec, PRL 2023]

# Summary and prospects

- For 3G detectors the mismatch error for semi-analytical models needs to be reduced by several orders of magnitude
- IMRIs are chalenging for existing modeling techniques and current templates are not reliable for $q \gtrsim 30$
- Post-adiabatic waveforms agree remarkably well with the results from full numerical relativity with $1 \leqslant q \leqslant 10$
- Second-order black hole perturbation theory will be used to model EMRIs, IMRIs and possibly comparable-mass systems
- Prospects in the near future:
  - Add the transition to plunge and merger
  - Inclusion of the black hole and secondary spin
  - Extension to generic eccentric and inclined orbits

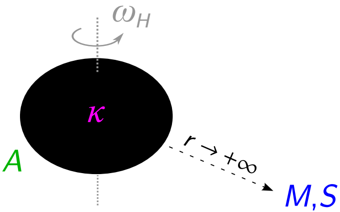
# Outline

# The black hole uniqueness theorem

- In 4D the only stationary vacuum black hole solution of the Einstein equation is the Kerr solution of mass $M$ and spin $S$

*"Black holes have no hair."* (J. A. Wheeler)

- Black hole event horizon $\mathcal{H}$ characterized by:
    - Angular velocity $\omega_H$
    - Surface gravity $\kappa$
    - Surface area $A$

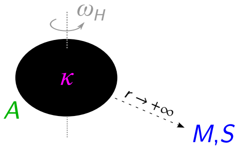



[Israel 1967; Carter 1971; Hawking 1973; Robinson 1975]

# The laws of black hole mechanics

- Zeroth law of mechanics:

$$\kappa = \text{const.} \quad (\text{on } \mathcal{H})$$

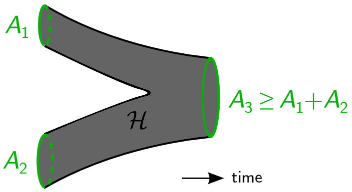



- First law of mechanics:

$$\delta M = \omega_H \, \delta S + \frac{\kappa}{8\pi} \, \delta A$$

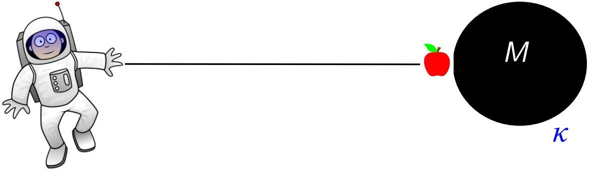
- Second law of mechanics:

$$\delta A \geq 0$$




[Hawking 1972; Bardeen, Carter & Hawking 1973]

# What is the horizon surface gravity?



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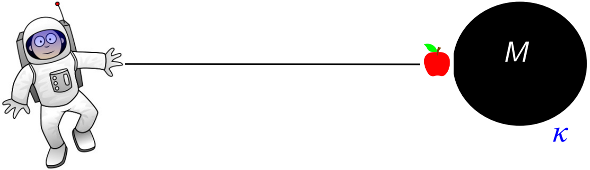


- For an event horizon $\mathcal{H}$ generated by a Killing field $k^a$:

$$\kappa^2 \equiv \frac{1}{2} \left( \nabla^a k^b \, \nabla_b k_a \right) \Big|_{\mathcal{H}}$$

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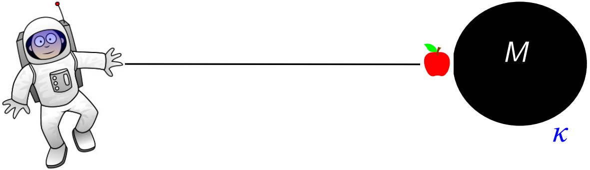


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- For a Schwarzschild black hole of mass $M$, this yields

$$\kappa = \frac{1}{4M} = \frac{GM}{R_S^2}$$

# Beyond stationary, isolated black holes

## Why?

- Astrophysical black holes are neither perfectly isolated, nor strictly stationary
- Of special interest are black holes that interact gravitationally with a companion in a compact binary system

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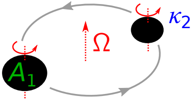
## How?

- Slowly evolving or dynamical horizons (quasi-local definitions)

✔ Physical setup that guarantees the existence of an isometry

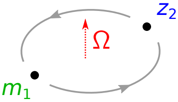
✔ Perturbative treatment of the problem: large separation, large mass ratio, weak tidal environment

# First law for circular-orbit compact binaries



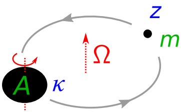

$$\delta M - \Omega\,\delta J = \sum_a 4\mu_a\kappa_a\,\delta\mu_a$$

CFC approximation
[Friedman *et al.* 2002]



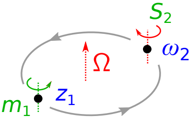

$$\delta M - \Omega\,\delta J = \sum_a z_a\,\delta m_a$$

PN approximation
[Le Tiec *et al.* 2012]




$$\delta M - \Omega\,\delta J = 4\mu\kappa\,\delta\mu + z\,\delta m$$

Perturbation theory
[Gralla & Le Tiec 2013]

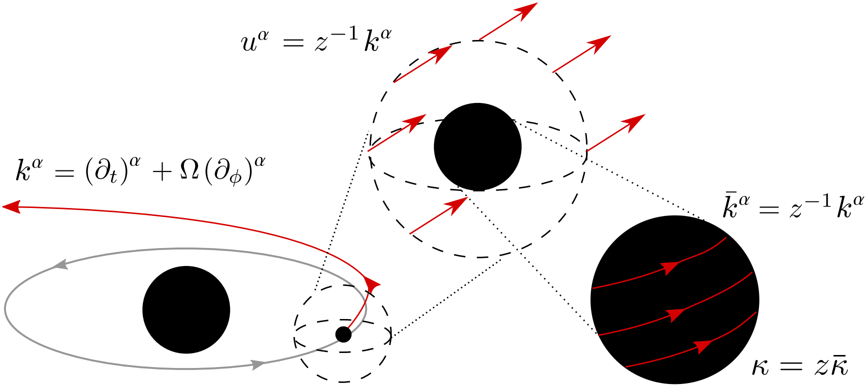



$$\delta M - \Omega\,\delta L = \sum_a \left(z_a\,\delta m_a + \omega_a\,\delta S_a\right)$$

ADM Hamiltonian
[Blanchet *et al.* 2013]

Helical isometry
[Ramond & Le Tiec 2022]

# Black hole surface gravity and redshift




[Zimmerman, Lewis & Pfeiffer, PRL 2016]

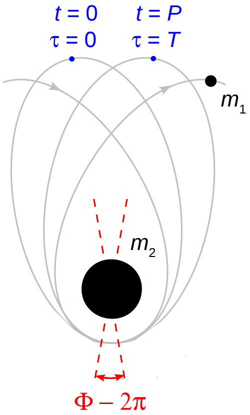
# Averaged redshift for eccentric orbits

- Generic eccentric orbit parameterized by the two requencies

$$\Omega_r = \frac{2\pi}{P}\,, \quad \Omega_\phi = \frac{\Phi}{P}$$

- Time average of redshift $z = \mathrm{d}\tau/\mathrm{d}t$ over one radial period

$$\langle z \rangle \equiv \frac{1}{P}\int_0^P z(t)\,\mathrm{d}t = \frac{1}{P}\int_0^{\mathcal{T}} \mathrm{d}\tau = \frac{\mathcal{T}}{P}$$




[Barack & Sago, PRD 2011]

# First law of mechanics for eccentric orbits

- Canonical ADM Hamiltonian $H(\mathbf{x}_a, \mathbf{p}_a; m_a)$ of two point particles with constant masses $m_a$
- Variation $\delta H$ + Hamilton's equation + orbital averaging:

$$\delta M = \Omega_\phi \, \delta L + \Omega_r \, \delta I_r + \sum_a \langle z_a \rangle \, \delta m_a$$

- Starting at 4PN order the binary dynamics gets nonlocal in time because of gravitational-wave tails:

$$H_{\text{tail}}^{\text{4PN}}[\mathbf{x}_a(t), \mathbf{p}_a(t)] = -\frac{GM}{5c^8} \, I_{ij}^{(3)}(t) \, \underset{2r}{\text{Pf}} \int_{-\infty}^{+\infty} \frac{\mathrm{d}\tau}{\tau} \, I_{ij}^{(3)}(t+\tau)$$

[Le Tiec, PRD 2015; Blanchet & Le Tiec, CQG 2017]

# Numerous applications of the first law

- Fix 'ambiguity parameters' in 4PN two-body equations of motion
  [Jaranowski & Schäfer 2012; Damour *et al.* 2014; Bernard *et al.* 2017]
- Inform the 5PN two-body Hamiltonian in a 'tutti-frutti' method
  [Bini, Damour & Geralico 2019; 2020]
- Calculate Schwarzschild and Kerr ISCO frequency shifts
  [Le Tiec *et al.* 2012; Akcay *et al.* 2012; Isoyama *et al.* 2014]
- Test cosmic censorship conjecture including GSF effects
  [Colleoni & Barack 2015; Colleoni *et al.* 2015]
- Calibrate EOB potentials in effective Hamiltonian
  [Barausse *et al.* 2012; Akcay & van de Meent 2016; Bini *et al.* 2016]
- Compare particle redshift to black hole surface gravity
  [Zimmerman, Lewis & Pfeiffer 2016; Le Tiec & Grandclément 2018]
- Benchmark for calculations of Schwarzschild IBCO frequency shift and gravitational binding energy [Barack *et al.* 2019; Pound *et al.* 2020]

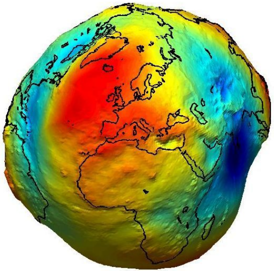
# Summary and prospects

- The classical laws of black hole mechanics can be extended to binary systems of compact objects
- Combined with the first law, the redshift $z(\Omega)$ provides crucial information about the binary dynamics:
  - Gravitational binding energy $E$ and angular momentum $J$
  - ISCO frequency $\Omega_{\text{ISCO}}$ and IBCO frequency $\Omega_{\text{IBCO}}$
  - EOB effective potentials $A$, $\bar{D}$, $Q$, ...
  - Horizon surface gravity $\kappa$
- Extensions in the near future:
  - Dissipative effects from radiation-reaction
  - Precessing spins and generic bound orbits
  - Finite-size effects from quadrupole moments
  - Unbound orbits and post-Minkowskian gravity

# Outline

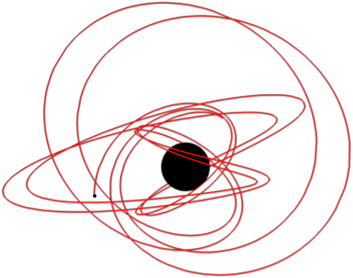
# Do isolated black holes have hair?

Geodesy



$M_{\ell,m}$ arbitrary

Botromeladesy

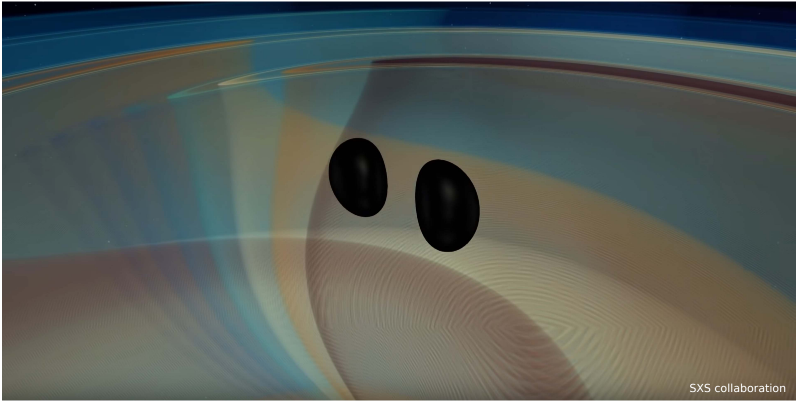


$$M_{\ell,0} + iS_{\ell,0} = M(ia)^{\ell}$$

**Objective:** test the black hole no-hair theorem of general relativity

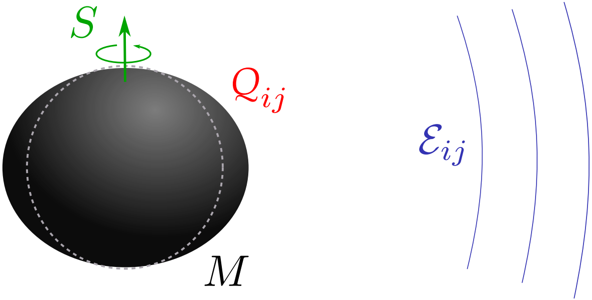
# Do tidally-interacting black holes deform?

Black hole **tomography** by gravitational-wave observations




**Objective:** measure the black hole tidal Love numbers with LISA

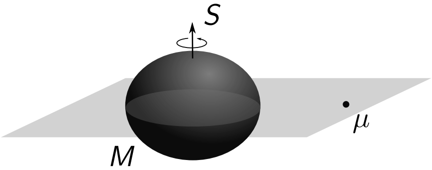
# Tidal deformability of Kerr black holes

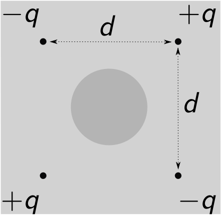



$$Q_{ij}^{\text{spin}} = -S_{\langle i}S_{j\rangle}/M \quad \text{and} \quad Q_{ij}^{\text{tidal}} = \tfrac{16}{45}\, M^3\, S^k\, \mathcal{E}^l{}_{(i}\,\epsilon_{j)kl}$$

[Le Tiec & Casals, PRL 2021; Le Tiec, Casals & Franzin, PRD 2021]

# Example: Newtonian static quadrupolar tide




$$\mathcal{E}_{ij} = \frac{\mu}{r^3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$Q_{ij}^{\text{tidal}} = 3Q\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\uparrow$$

$$\tfrac{8}{45}M^3 S\,\mu/r^3 = qd^2$$

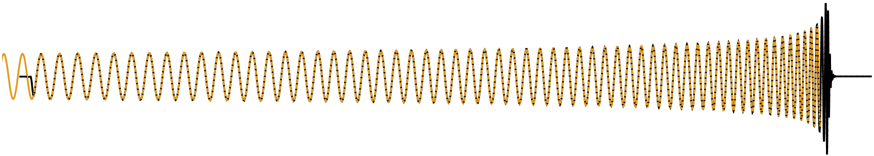
[Le Tiec & Casals, PRL 2021; Le Tiec, Casals & Franzin, PRD 2021]

# A burst of activity on BH tidal deformability

- Other backgrounds, generic spin-$s$ fields and higher dimensions
  [Hui *et al.*, JCAP 2021; Pereñiguez & Cardoso, PRD 2022; Rodriguez *et al.*, PRD 2023; Charalambous & Ivanov, JHEP 2023; Charalambous, JHEP 2024]
- Dissipative nature of Kerr black hole tidal deformability
  [Chia, PRD 2021; Goldberger *et al.*, JHEP 2021; Charalambous, JHEP 2021; Prasad Bhatt *et al.*, PRD 2023]
- Hidden symmetry and vanishing black hole Love numbers
  [Charalambous *et al.*, PRL 2021; Hui *et al.*, JCAP 2022; Charalambous *et al.*, JHEP 2022; Achour *et al.*, JHEP 2022; Hui *et al.*, JHEP 2022; Berens *et al.*, JCAP 2023; Katagiri *et al.*, PRD 2023; Rai & Santoni 2024]
- Scattering amplitudes and vanishing black hole Love numbers
  [Creci *et al.*, PRD 2021; Ivanov & Zhou, PRL 2023; Saketh *et al.*, PRD 2024]
- Effective Field Theory, matching and logarithmic corrections
  [Ivanov & Zhou, PRD 2023]
- Nonlinearities in the tidal Love numbers of black holes
  [De Luca *et al.*, PRD 2023; Maria Riva *et al.* 2023; Hadad *et al.* 2024]

# Summary and prospects

- Program of black hole tomography by gravitational-wave observations
- Spinning black holes deform like any other self-gravitating body, despite being particularly "rigid" compact objects
- New black hole test of the Kerr-like nature of the massive compact objects at the center of galaxies?
- Future research directions:
  - Relation between tidal deformability and horizon viscosity
  - Compute Kerr black hole shape tidal Love numbers
  - Explore link between source and field multipoles



**Thank you for your attention!**