

# Tidal deformations of black holes

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# TIDES

Low tide

High tide



High tide



Moon

Low tide

① Motivations

② Newtonian tides

③ Relativistic tides

④ Black hole Love numbers

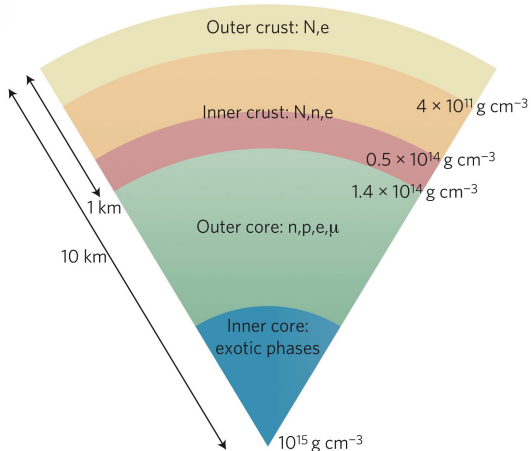
① Motivations

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③ Relativistic tides

④ Black hole Love numbers

# What is the internal structure of a neutron star?

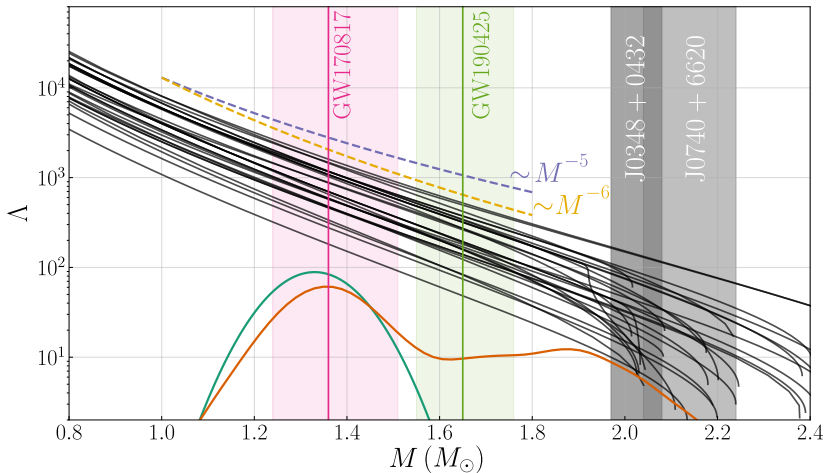


GW observations as probes of **neutron star equation of state**

# First constraints from GW observations

[Chatziioannou, GRG 2020]

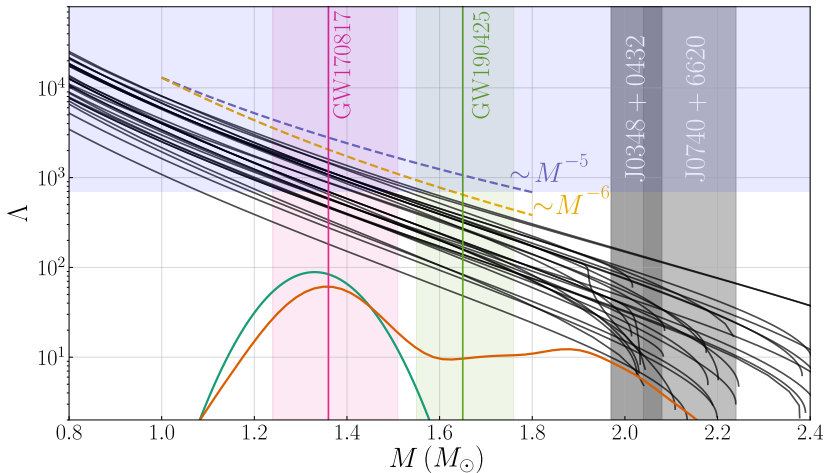
$$\Lambda \equiv \lambda_2/M^5 \propto k_2/C^5$$



# First constraints from GW observations

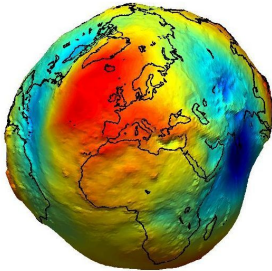
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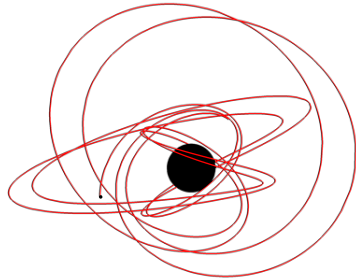
# Do isolated black holes have hair?

Geodesy



$M_{\ell,m}$  arbitrary

Botromeladesy

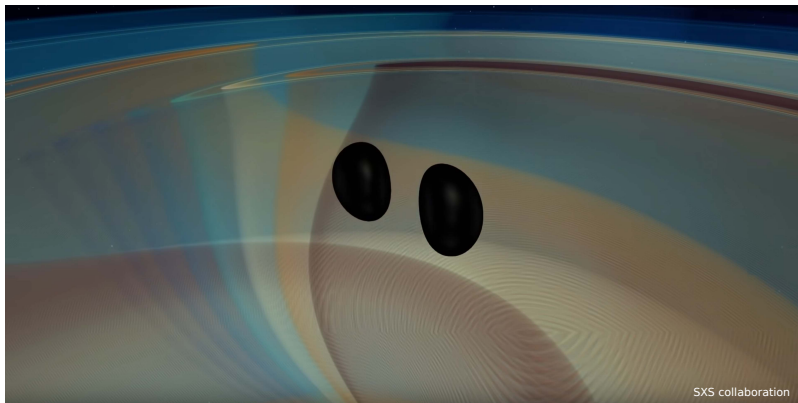


$M_{\ell,0} + iS_{\ell,0} = M(ia)^\ell$

**Objective:** test the black hole no-hair theorem of general relativity

## Do tidally-interacting black holes deform?

Black hole **tomography** by gravitational-wave observations

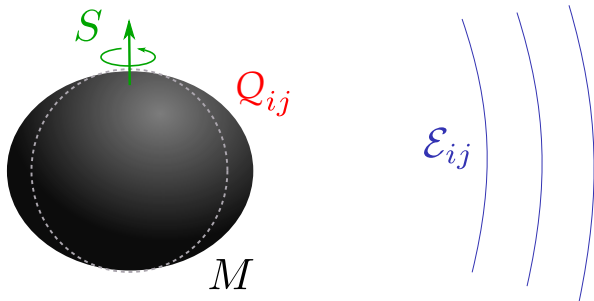


SXS collaboration

**Objective:** measure the black hole tidal Love numbers with LISA

# Tidal deformability of Kerr black holes

[Le Tiec, Casals & Franzin, PRD 2021]



$$Q_{ij}^{\text{spin}} = -S_{\langle i} S_{j \rangle} / M \quad \text{and} \quad Q_{ij}^{\text{tidal}} = \frac{16}{45} M^3 S^k \mathcal{E}^l{}_{(i} \epsilon_{j)kl}$$

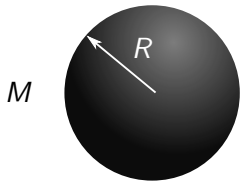
① Motivations

② Newtonian tides

③ Relativistic tides

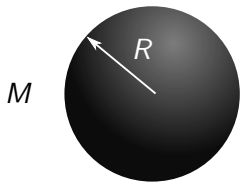
④ Black hole Love numbers

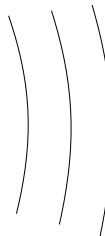
## Newtonian theory of Love numbers



$$U = \frac{M}{r}$$

## Newtonian theory of Love numbers



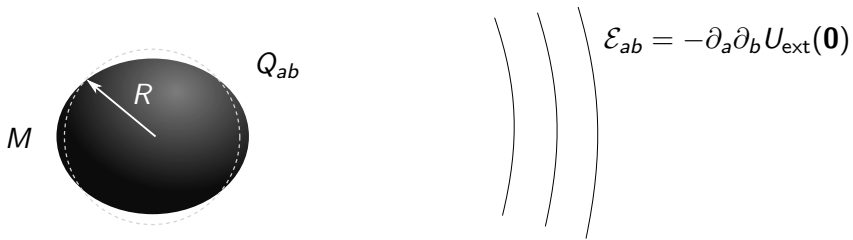


Three vertical, slightly curved lines representing external potential. To their right is the equation  $\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$ .

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

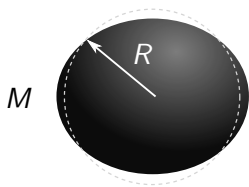
$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab}$$

## Newtonian theory of Love numbers

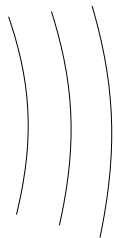


$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} + \frac{3}{2} \frac{x^a x^b Q_{ab}}{r^5}$$

## Newtonian theory of Love numbers



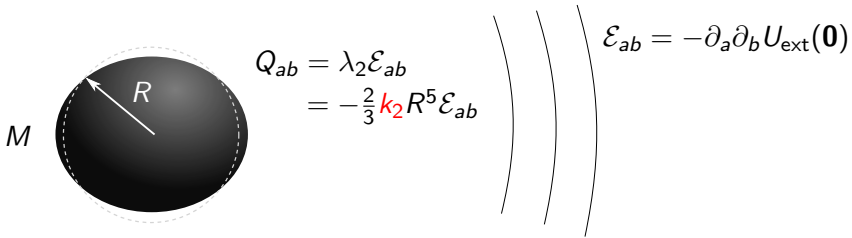
$$Q_{ab} = \lambda_2 \mathcal{E}_{ab}$$



$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

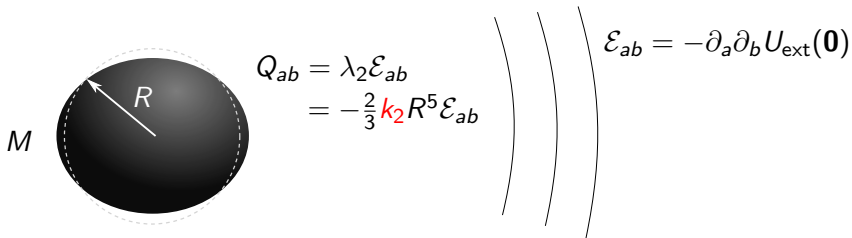
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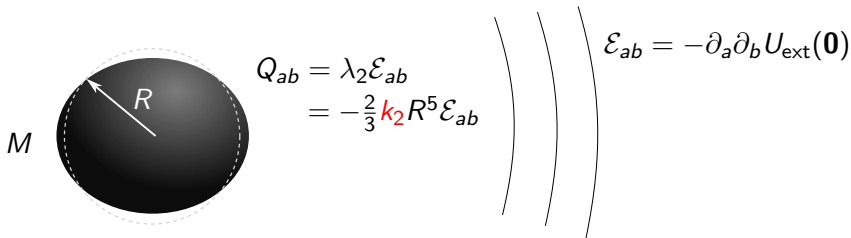
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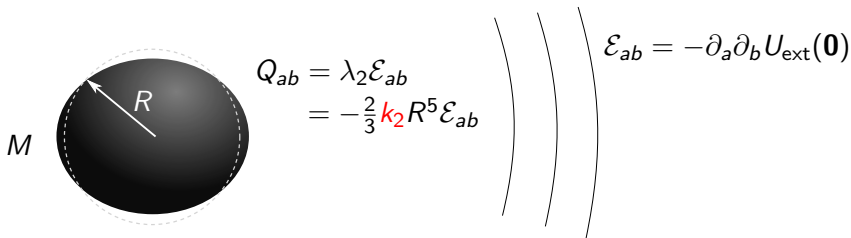
$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} \left[ 1 + 2k_2 \left( \frac{R}{r} \right)^5 \right]$$

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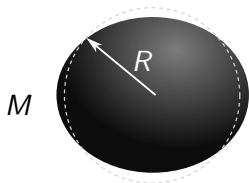
$$U = \frac{M}{r} - \sum_{\ell \geq 2} \frac{(\ell - 2)!}{\ell!} x^{a_1} \dots x^{a_\ell} \mathcal{E}_{a_1 \dots a_\ell} \left[ 1 + 2k_\ell \left( \frac{R}{r} \right)^{2\ell+1} \right]$$

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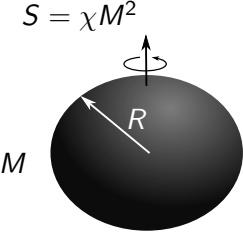


$$\begin{aligned}
 Q_{ab} &= \lambda_2 \mathcal{E}_{ab} \\
 &= -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab}
 \end{aligned}$$

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

$$\psi_0 = \sum_{\ell \geq 2} \sum_{|m| \leq \ell} \sqrt{\frac{(\ell+2)(\ell+1)}{\ell(\ell-1)}} r^{\ell-2} \mathcal{E}_{\ell m} \left[ 1 + 2k_\ell \left( \frac{R}{r} \right)^{2\ell+1} \right] {}_2Y_{\ell m}$$

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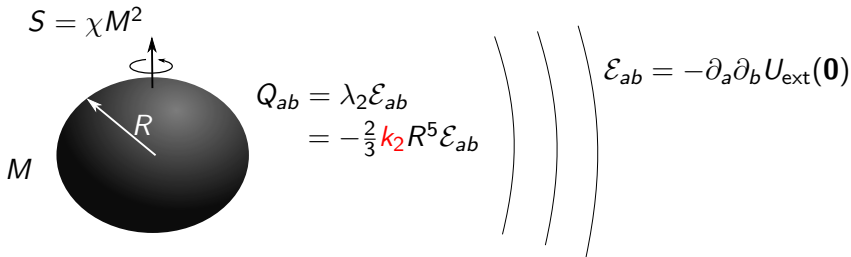
$S = \chi M^2$   
 $M$   
 $R$

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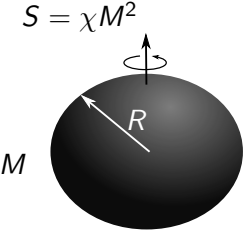
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$$k_{\ell m} = k_{\ell}^{(0)} + im\chi k_{\ell}^{(1)} + O(\chi^2)$$

## Newtonian theory of Love numbers



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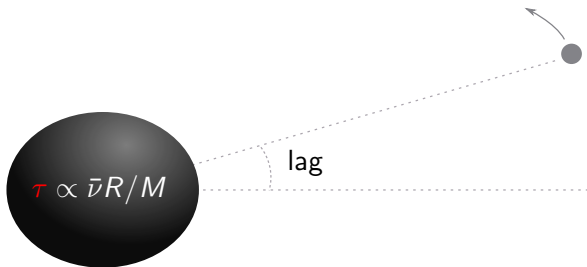
$$Q_{ab} = \lambda_2 \mathcal{E}_{ab} = -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab}$$

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Tidal Love numbers  $k_{\ell m}$   $\longleftrightarrow$  **body's internal structure**

## Tidal dissipation: lag, heating and torquing



$$\begin{aligned} Q_{ab}(t) &= -\frac{2}{3} k_2 R^5 [\mathcal{E}_{ab}(t) - \tau \dot{\mathcal{E}}_{ab}(t) + \dots] \\ &= -\frac{2}{3} k_2 R^5 [\mathcal{E}_{ab}(t - \tau) + \dots] \end{aligned}$$

① Motivations

② Newtonian tides

③ Relativistic tides

④ Black hole Love numbers

## Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_L \propto [C_{0a_1 0a_2; a_3 \dots a_\ell}]^{\text{STF}} \quad \text{and} \quad \mathcal{B}_L \propto [\varepsilon_{a_1 bc} C_{a_2 0bc; a_3 \dots a_\ell}]^{\text{STF}}$$

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- Metric and Geroch-Hansen multipole moments:

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- Two families of tidal deformability parameters:

$$\delta M_L = \lambda_\ell^{\text{el}} \mathcal{E}_L \quad \text{and} \quad \delta S_L = \lambda_\ell^{\text{mag}} \mathcal{B}_L$$

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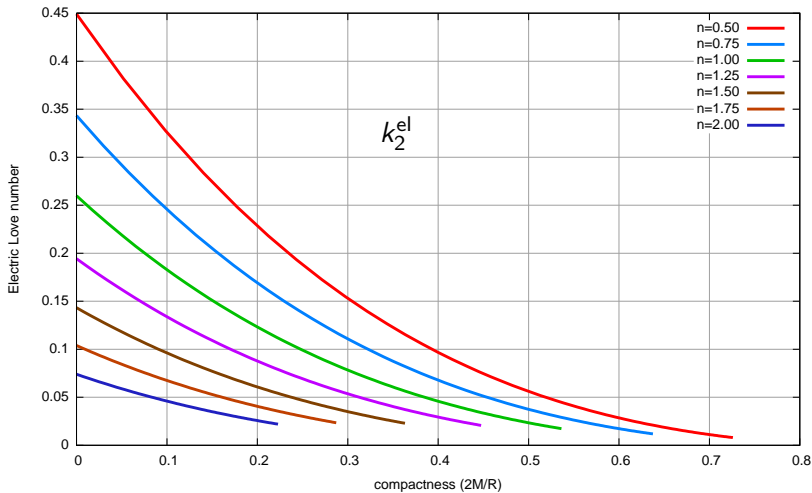
$$\delta M_L = \lambda_\ell^{\text{el}} \mathcal{E}_L \quad \text{and} \quad \delta S_L = \lambda_\ell^{\text{mag}} \mathcal{B}_L$$

- Dimensionless tidal Love numbers:

$$k_\ell^{\text{el/mag}} \equiv -\frac{(2\ell - 1)!!}{2(\ell - 2)!} \frac{\lambda_\ell^{\text{el/mag}}}{R^{2\ell+1}}$$

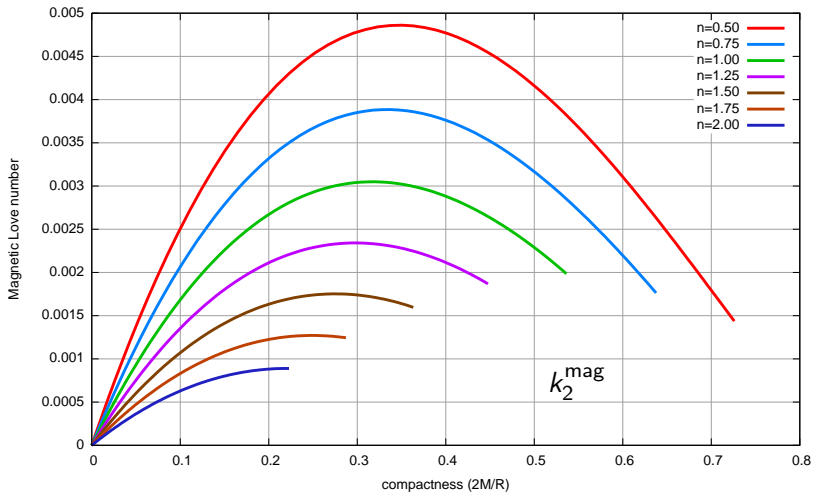
# Love numbers of neutron stars

[Binnington & Poisson, PRD 2009]



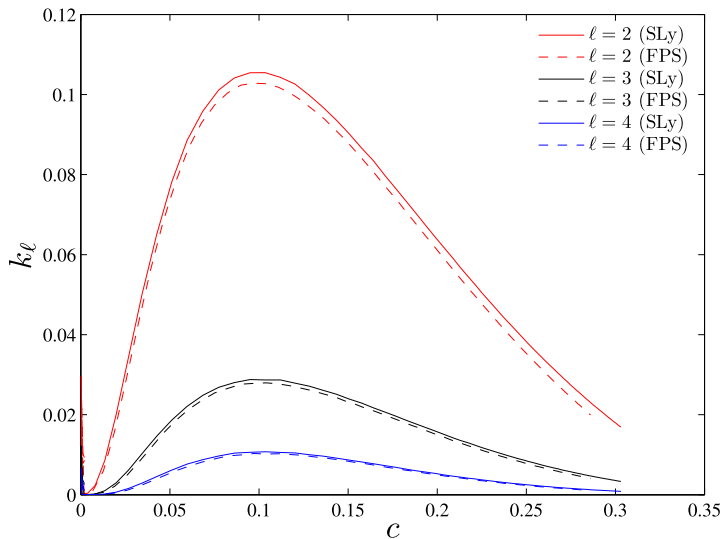
# Love numbers of neutron stars

[Binnington & Poisson, PRD 2009]



# Love numbers of neutron stars

[Damour & Nagar, PRD 2009]



## Love numbers of *spinning* compact objects

- The spin breaks the spherical symmetry of the background
  - No proportionality between  $(\delta M_L, \delta S_L)$  and  $(\mathcal{E}_L, \mathcal{B}_L)$
  - Degeneracy of the azimuthal number  $m$  lifted
  - Parity mixing and mode couplings allowed

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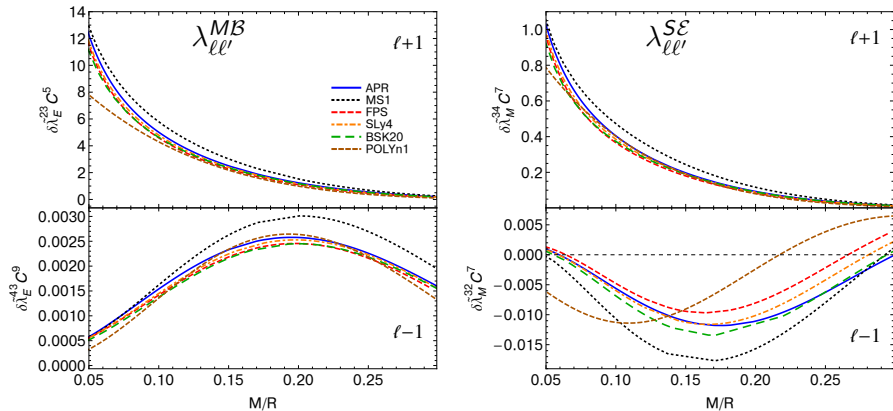
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- Four families of tidal deformability parameters:

$$\begin{aligned} \lambda_{\ell\ell'm}^{M\mathcal{E}} &\equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell'm}} & \lambda_{\ell\ell'm}^{S\mathcal{B}} &\equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell'm}} \\ \lambda_{\ell\ell'm}^{S\mathcal{E}} &\equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{E}_{\ell'm}} & \lambda_{\ell\ell'm}^{M\mathcal{B}} &\equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{B}_{\ell'm}} \end{aligned}$$

# Love numbers of *spinning* neutron stars

[Pani, Gualtieri & Ferrari, PRD 2015]



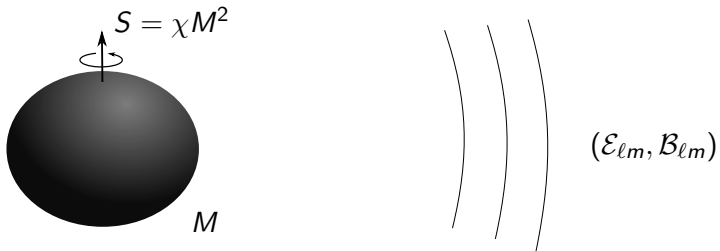
Restricted to an *axisymmetric* tidal perturbation ( $m = 0$ )

- ① Motivations
- ② Newtonian tides
- ③ Relativistic tides
- ④ Black hole Love numbers

## Do black holes have zero Love numbers?

| Reference                   | Background       | Tidal field                |
|-----------------------------|------------------|----------------------------|
| [Binnington & Poisson 2009] | Schwarzschild    | weak, generic $\ell$       |
| [Damour & Nagar 2009]       | Schwarzschild    | weak, generic $\ell$       |
| [Kol & Smolkin 2012]        | Schwarzschild    | weak, electric-type        |
| [Chakrabarti et al. 2013]   | Schwarzschild    | weak, electric, $\ell = 2$ |
| [Gürlebeck 2015]            | Schwarzschild    | strong, axisymmetric       |
| [Landry & Poisson 2015]     | Kerr to $O(S)$   | weak, quadrupolar          |
| [Pani et al. 2015]          | Kerr to $O(S^2)$ | weak, $(\ell, m) = (2, 0)$ |
| [Le Tiec & Casals 2021]     | Exact Kerr       | weak, generic $\ell$       |
| [Chia 2021]                 | Exact Kerr       | weak, generic $\ell$       |
| [Goldberger et al. 2021]    | Exact Kerr       | weak, generic $\ell$       |
| [Charalambous et al. 2021]  | Exact Kerr       | weak, generic $\ell$       |
| [Hui 2021]                  | Schwar-(A)dS     | weak, generic $\ell$       |

## Investigating Kerr's Love



$$(\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}) \rightarrow \psi_0 \rightarrow \Psi \rightarrow h_{\alpha\beta} \rightarrow (M_{\ell m}, S_{\ell m}) \rightarrow \lambda_{\ell m}^{M/S, \mathcal{E}/\mathcal{B}}$$

Metric reconstruction through the Hertz potential  $\Psi$

## Perturbed Weyl scalar

- Recall that in the Newtonian limit we established

$$\lim_{c \rightarrow \infty} \psi_0^{\ell m} \propto \mathcal{E}_{\ell m} r^{\ell-2} [1 + 2k_{\ell m} (R/r)^{2\ell+1}] {}_2Y_{\ell m}(\theta, \phi)$$

- For a Kerr black hole the perturbed Weyl scalar reads

$$\psi_0^{\ell m} \propto [\mathcal{E}_{\ell m} + \frac{3i}{\ell+1} \mathcal{B}_{\ell m}] R_{\ell m}(r) {}_2Y_{\ell m}(\theta, \phi)$$

- Asymptotic behavior of general solution of static radial Teukolsky equation:

$$R_{\ell m}(r) = \underbrace{r^{\ell-2} (1 + \dots)}_{\text{tidal field } R_{\ell m}^{\text{tidal}}} + k_{\ell m} \underbrace{r^{-\ell-3} (1 + \dots)}_{\text{linear response } R_{\ell m}^{\text{resp}}}$$

## Why analytic continuation?

$$R_{\ell m}(r) = \underbrace{r^{\ell-2} (1 + \dots)}_{\text{tidal field } R_{\ell m}^{\text{tidal}}} + k_{\ell m} \underbrace{r^{-\ell-3} (1 + \dots)}_{\text{linear response } R_{\ell m}^{\text{resp}}}$$

Ambiguity in the linear response [Fang & Lovelace 2005; Gralla 2018]

The decaying solution  $R_{\ell m}^{\text{resp}}$  is affected by a radial coord. transfo.

Ambiguity in the tidal field [Pani, Gualtieri, Maselli & Ferrari 2015]

The growing solution  $R_{\ell m}^{\text{tidal}} + \alpha R_{\ell m}^{\text{resp}}$  still qualifies as a tidal solution

## Kerr black hole linear response

$$R_{\ell m}(r) = \underbrace{R_{\ell m}^{\text{tidal}}(r)}_{\sim r^{\ell-2}} + 2k_{\ell m} \underbrace{R_{\ell m}^{\text{resp}}(r)}_{\sim r^{-(\ell+3)}}$$

- The coefficients  $k_{\ell m}$  can be interpreted as the Newtonian Love numbers of a Kerr black hole and read

$$k_{\ell m} = -im\chi \frac{(\ell+2)!(\ell-2)!}{4(2\ell+1)!(2\ell)!} \prod_{n=1}^{\ell} [n^2(1-\chi^2) + m^2\chi^2]$$

- The linear response vanishes identically when:
  - the black hole spin vanishes ( $\chi = 0$ )
  - the tidal field is axisymmetric ( $m = 0$ )
- Reconstruct the Kerr black hole response  $h_{\alpha\beta}^{\text{resp}}$  via  $\Psi^{\text{resp}}$

## Love numbers of a Kerr black hole

- We compute the Love numbers to **linear** order in  $\chi \equiv S/M^2$

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$$\delta M_{2m} \doteq \frac{im\chi}{180} (2M)^5 \mathcal{E}_{2m} \quad \text{and} \quad \delta S_{2m} \doteq \frac{im\chi}{180} (2M)^5 \mathcal{B}_{2m}$$

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- The black hole tidal bulge is **rotated by  $45^\circ$**  with respect to the quadrupolar tidal perturbation
- The associated dimensionless tidal Love numbers are

$$k_{2m}^{ME} = k_{2m}^{SB} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{MB} = k_{2m}^{SE} \doteq 0$$

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$$\delta M_{2m} \doteq \frac{im\chi}{180} (2M)^5 \mathcal{E}_{2m} \quad \text{and} \quad \delta S_{2m} \doteq \frac{im\chi}{180} (2M)^5 \mathcal{B}_{2m}$$

- The black hole tidal bulge is **rotated by  $45^\circ$**  with respect to the quadrupolar tidal perturbation
- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} \doteq 0$$

- For a dimensionless black hole spin  $\chi = 0.1$  this gives

$$|k_{2,\pm 2}| \simeq 2 \times 10^{-3} \quad \longrightarrow \quad \text{black holes are “rigid”}$$

## Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\text{el}} \mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\text{mag}} \mathcal{B}_{ab}$$

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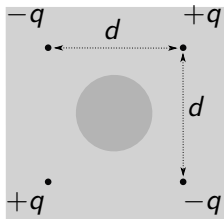
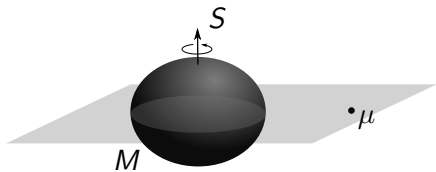
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- To *linear order* in the black hole spin vector  $S^a$  we find

$$\delta M_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{E}^d_{(a} \mathcal{E}_{b)cd}$$
$$\delta S_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{B}^d_{(a} \mathcal{E}_{b)cd}$$

## Newtonian static quadrupolar tide



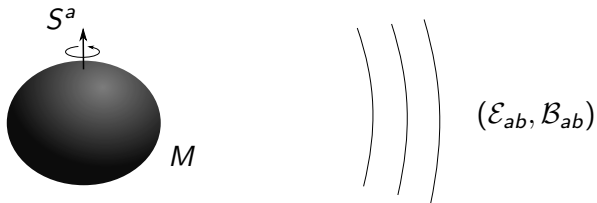
$$\mathcal{E}_{ab} = \frac{\mu}{r^3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\delta M_{ab} \doteq 3Q \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

↑

$$\frac{\chi}{180} (2M)^5 \frac{\mu}{r^3} = qd^2$$

## Tidal torquing of a spinning black hole



- Any spinning body interacting with a tidal environment suffers a **tidal torquing** [Thorne & Hartle 1980]

$$\langle \dot{S}^a \rangle = -\epsilon^{abc} \langle M_{bd} \mathcal{E}^d_c + S_{bd} \mathcal{B}^d_c \rangle$$

- Applied to a **spinning black hole** this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi \left[ 2 \langle \mathcal{E}^{ab} \mathcal{E}_{ab} \rangle - 3 \langle \mathcal{E}_{ab} S^b \mathcal{E}^{ac} S_c \rangle + (\mathcal{E} \rightarrow \mathcal{B}) \right]$$

- Full agreement with independent calculation by [Poisson 2004]

# A burst of activity on BH tidal deformability

- Other **backgrounds**, generic **spin- $s$**  fields and higher **dimensions**  
[Hui *et al.*, JCAP 2021; Pereñíguez & Cardoso, PRD 2022; Rodriguez *et al.*, PRD 2023; Charalambous & Ivanov, JHEP 2023; Charalambous, JHEP 2024]
- **Dissipative nature** of Kerr black hole tidal deformability  
[Chia, PRD 2021; Goldberger *et al.*, JHEP 2021; Charalambous, JHEP 2021; Prasad Bhatt *et al.*, PRD 2023]
- **Hidden symmetry** and vanishing black hole Love numbers  
[Charalambous *et al.*, PRL 2021; Hui *et al.*, JCAP 2022; Charalambous *et al.*, JHEP 2022; Achour *et al.*, JHEP 2022; Hui *et al.*, JHEP 2022; Berens *et al.*, JCAP 2023; Katagiri *et al.*, PRD 2023; Rai & Santoni 2024]
- **Scattering amplitudes** and vanishing black hole Love numbers  
[Creci *et al.*, PRD 2021; Ivanov & Zhou, PRL 2023; Saketh *et al.*, PRD 2024]
- Effective Field Theory, matching and **logarithmic corrections**  
[Ivanov & Zhou, PRD 2023]
- **Nonlinearities** in the tidal Love numbers of black holes  
[De Luca *et al.*, PRD 2023; Maria Riva *et al.* 2023; Hadad *et al.* 2024; Combaluzier-Szteinsznaider *et al.* 2024; Kehagias & Riotto 2024]

## Summary

- Love numbers of Kerr black holes **do not vanish** in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- **Spinning black holes deform** like any other self-gravitating body, despite being particularly “rigid” compact objects
- This is closely related to the phenomenon of **tidal torquing**
- **New black hole test** of the Kerr-like nature of the massive compact objects at the center of galaxies?

**Spinning black holes fall in Love!**