Spinning black holes fall in Love

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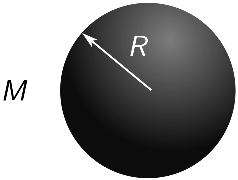
Collaborators: M. Casals & E. Franzin

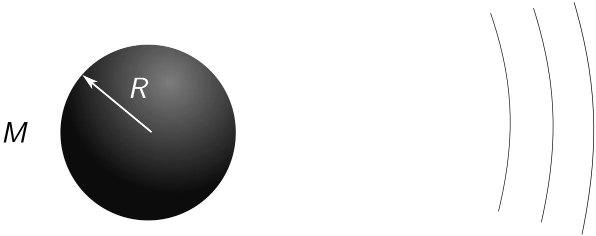
Submitted, arXiv:2007.00214 [gr-qc]

Newtonian theory of Love numbers



$$U = \frac{M}{r}$$

Newtonian theory of Love numbers



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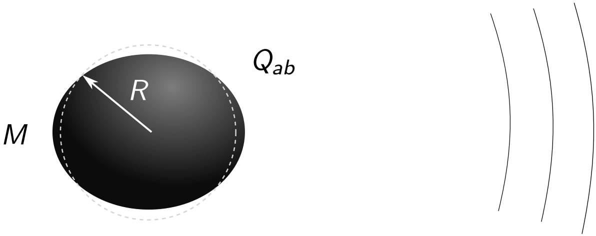

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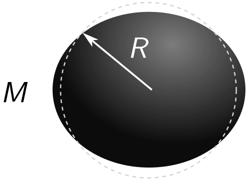
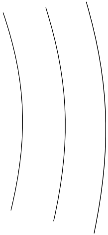


$$Q_{ab} = \lambda_2 \mathcal{E}_{ab}$$



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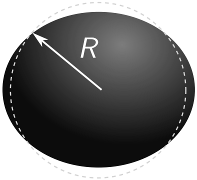

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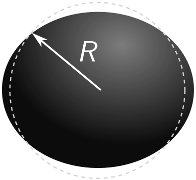

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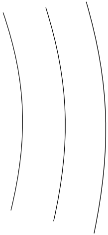

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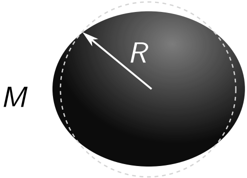

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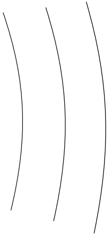

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$$U = \frac{M}{r} - \sum_{\ell \geqslant 2} \frac{(\ell-2)!}{\ell!} \, x^{a_1 \cdots a_\ell} \mathcal{E}_{a_1 \cdots a_\ell} \left[1 + 2 k_\ell \left(\frac{R}{r} \right)^{2\ell+1} \right]$$

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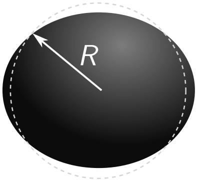

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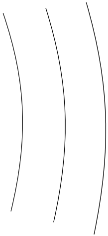

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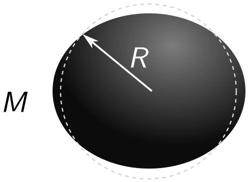

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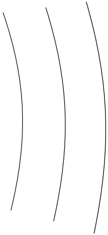

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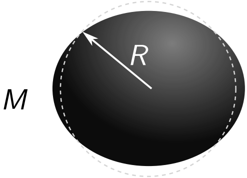

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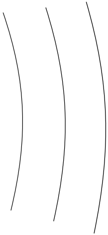

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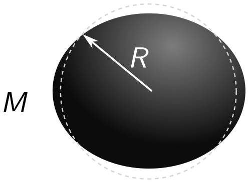

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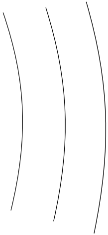

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Tidal Love numbers $k_{\ell m} \longleftrightarrow$ **body's internal structure**

Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_{a_1\cdots a_\ell} \propto (C_{0a_10a_2;a_3\cdots a_\ell})_{\mathsf{STF}} \quad \text{and} \quad \mathcal{B}_{a_1\cdots a_\ell} \propto (\varepsilon_{a_1bc}C_{a_20bc;a_3\cdots a_\ell})_{\mathsf{STF}}$$

- Metric and Geroch-Hansen multipole moments:

$$g_{\alpha\beta} = \mathring{g}_{\alpha\beta} + \underbrace{h^{\text{tidal}}_{\alpha\beta}}_{\sim r^\ell} + \underbrace{h^{\text{resp}}_{\alpha\beta}}_{\sim r^{-(\ell+1)}} \quad \longrightarrow \quad \begin{cases} M_{\ell m} = \mathring{M}_{\ell m} + \delta M_{\ell m} \\ S_{\ell m} = \mathring{S}_{\ell m} + \delta S_{\ell m} \end{cases}$$

- Four families of tidal deformability parameters:

$$\lambda^{M\mathcal{E}}_{\ell m} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell m}} \qquad \lambda^{S\mathcal{B}}_{\ell m} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell m}}$$

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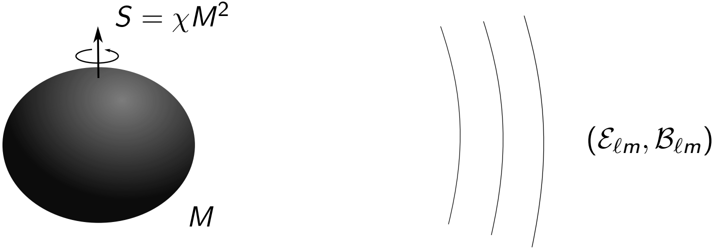
Black holes have *zero* Love numbers

Reference	**Background**	**Tidal field**
[Binnington & Poisson 2009]	Schwarzschild	weak, generic ℓ
[Damour & Nagar 2009]	Schwarzschild	weak, generic ℓ
[Kol & Smolkin 2012]	Schwarzschild	weak, electric-type
[Chakrabarti et al. 2013]	Schwarzschild	weak, electric, $\ell = 2$
[Gürlebeck 2015]	Schwarzschild	strong, axisymmetric
[Landry & Poisson 2015]	Kerr to $O(S)$	weak, quadrupolar
[Pani et al. 2015]	Kerr to $O(S^2)$	weak, $(\ell, m) = (2, 0)$

Problem of **fine-tuning** from an Effective-Field-Theory perspective

Investigating Kerr's Love



$$(\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}) \to \psi_0 \to \Psi \to h_{\alpha\beta} \to (M_{\ell m}, S_{\ell m}) \to \lambda_{\ell m}^{M/S, \mathcal{E}/\mathcal{B}}$$

Metric reconstruction through the Hertz potential Ψ

Love numbers of a Kerr black hole

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- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} = -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} = 0$$

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- For a dimensionless black hole spin $\chi = 0.1$ this gives

$$|k_{2,\pm 2}| \simeq 2 \times 10^{-3} \quad \longrightarrow \quad \text{black holes are } \textbf{“stiff”}$$

Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\mathrm{el}} \mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\mathrm{mag}} \mathcal{B}_{ab}$$

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- For a spinning black hole we have the more general tensorial relations

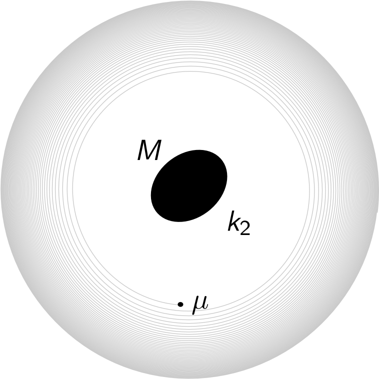
$$\delta M_{ab} = \lambda_{abcd} \mathcal{E}_{cd} \quad \text{and} \quad \delta S_{ab} = \lambda_{abcd} \mathcal{B}_{cd}$$

where

$$\lambda_{abcd} = \frac{\chi}{180} (2M)^5 \begin{pmatrix} \mathbf{I}_{11} & \mathbf{I}_{12} & \mathbf{I}_{13} \\ \mathbf{I}_{12} & -\mathbf{I}_{11} & \mathbf{I}_{23} \\ \mathbf{I}_{13} & \mathbf{I}_{23} & \mathbf{0} \end{pmatrix}, \quad \mathbf{I}_{11} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Observing black hole tidal deformability

[Pani & Maselli, IJMPD 2019]



Accumulated GW phase in LISA band during quasi-circular inspiral down to Schwarzschild ISCO:

$$\Phi_{\rm tidal} \simeq -80 \left(\frac{10^{-7}}{\mu/M}\right)\left(\frac{k_2}{0.002}\right)$$

↑

like 1st order dissipative self-force

Summary

- Love numbers of Kerr black holes do not vanish in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- Kerr black holes deform like any other self-gravitating body, despite being particularly “stiff” compact objects
- The black hole tidal deformation contribution to the GW phase of EMRIs could be detectable by LISA
- New black hole test of the Kerr-like nature of the massive compact objects at the center of galaxies

Spinning black holes fall in Love!