

# Spinning black holes fall in Love

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arXiv:2007.00214 and arXiv:2010.15795

# TIDES

Low tide

High tide



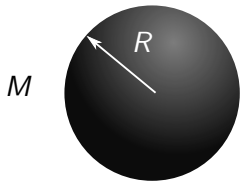
High tide



Moon

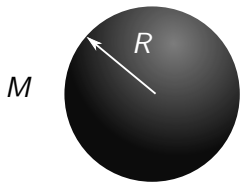
Low tide

## Newtonian theory of tidal Love numbers



$$U = \frac{M}{r}$$

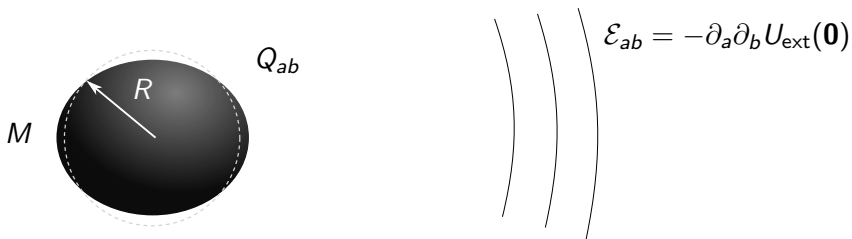
## Newtonian theory of tidal Love numbers



Three vertical, slightly curved lines representing tidal deformation. To the right of these lines is the equation  $\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$ .

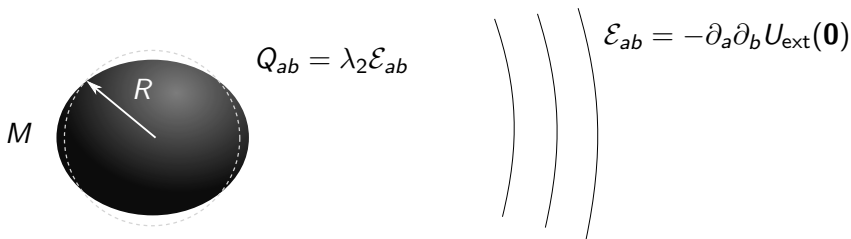
$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab}$$

## Newtonian theory of tidal Love numbers



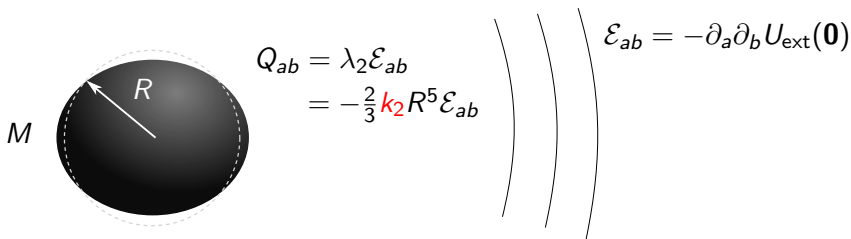
$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} + \frac{3}{2} \frac{x^a x^b Q_{ab}}{r^5}$$

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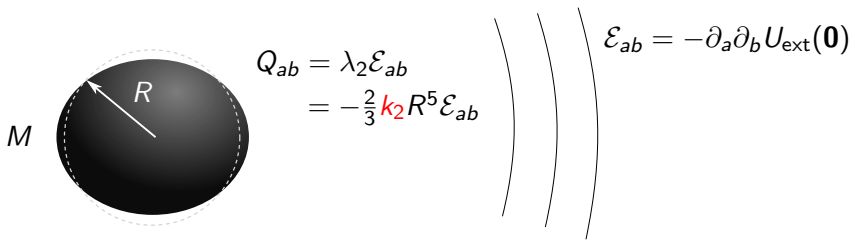
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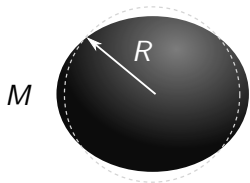
## Newtonian theory of tidal Love numbers



$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} \left[ 1 + 2k_2 \left( \frac{R}{r} \right)^5 \right]$$



## Newtonian theory of tidal Love numbers

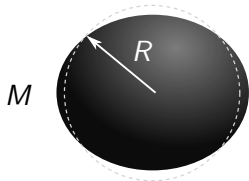


$$Q_{ab} = \lambda_2 \mathcal{E}_{ab} \\ = -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab}$$

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$$U = \frac{M}{r} - \sum_{\ell \geq 2} \frac{(\ell-2)!}{\ell!} x^{a_1} \dots x^{a_\ell} \mathcal{E}_{a_1 \dots a_\ell} \left[ 1 + 2k_\ell \left( \frac{R}{r} \right)^{2\ell+1} \right]$$

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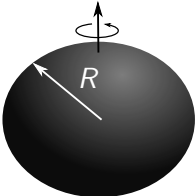
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$M$

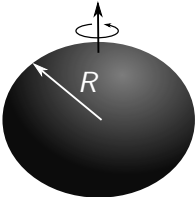
$R$

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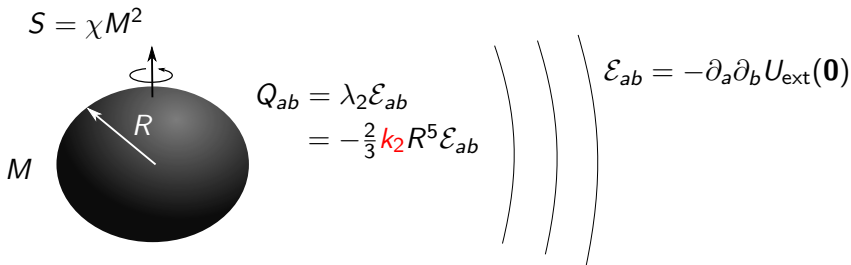
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$$k_{\ell m} = k_\ell^{(0)} + im\chi k_\ell^{(1)} + O(\chi^2)$$

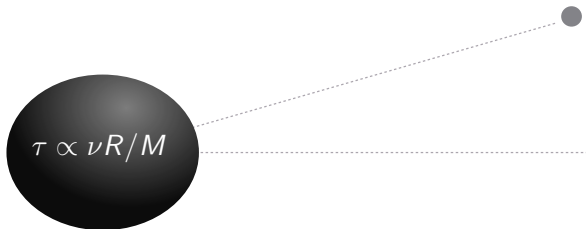
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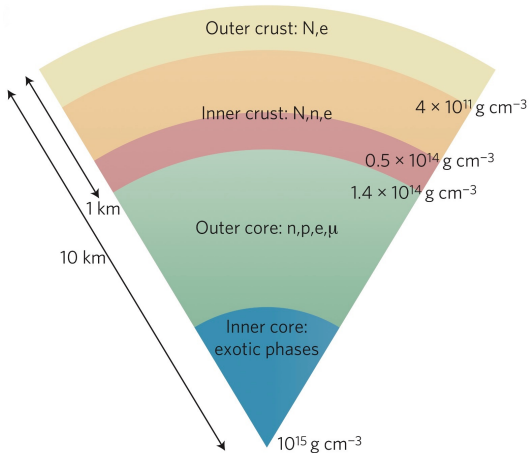
Tidal Love numbers  $k_{\ell m}$   $\longleftrightarrow$  **body's internal structure**

## Tidal dissipation: lag, heating and torquing



$$\begin{aligned} Q_{ab}(t) &= -\frac{2}{3}k_2R^5 [\mathcal{E}_{ab}(t) - \tau\dot{\mathcal{E}}_{ab}(t) + \dots] \\ &= -\frac{2}{3}k_2R^5 [\mathcal{E}_{ab}(t - \tau) + \dots] \end{aligned}$$

# Internal structure of neutron stars

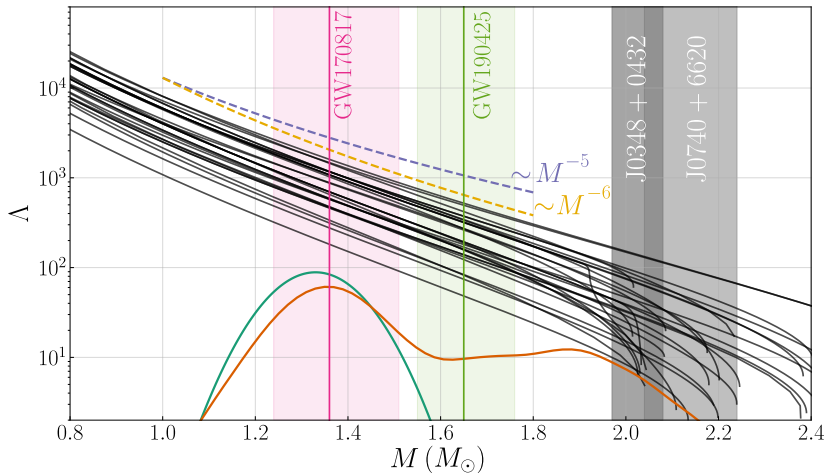


GW observations as probes of **neutron star internal structure**

# Gravitational-wave observations

[Chatziioannou, GRG 2020]

$$\Lambda \equiv \lambda_2/M^5 \propto k_2/C^5$$

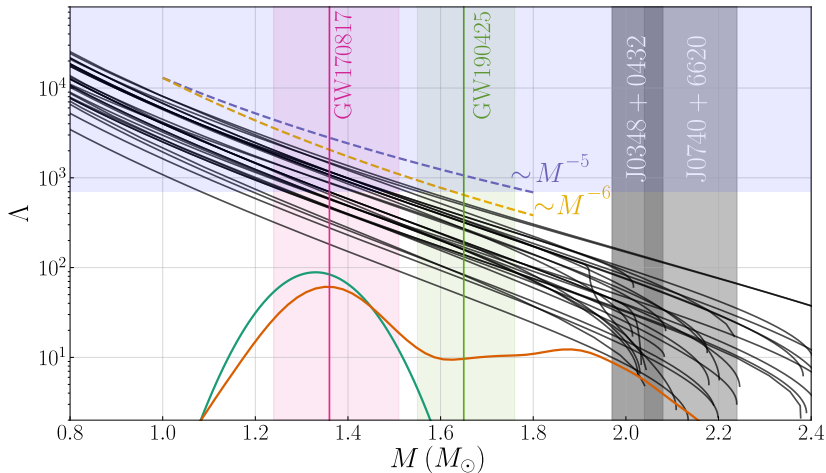




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## Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_L \propto [C_{0a_1 0a_2; a_3 \dots a_\ell}]^{\text{STF}} \quad \text{and} \quad \mathcal{B}_L \propto [\varepsilon_{a_1 bc} C_{a_2 0bc; a_3 \dots a_\ell}]^{\text{STF}}$$

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$$g_{\alpha\beta} = \dot{g}_{\alpha\beta} + \underbrace{h_{\alpha\beta}^{\text{tidal}}}_{\sim r^\ell} + \underbrace{h_{\alpha\beta}^{\text{resp}}}_{\sim r^{-(\ell+1)}} \quad \longrightarrow \quad \begin{cases} M_L = \dot{M}_L + \delta M_L \\ S_L = \dot{S}_L + \delta S_L \end{cases}$$

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- Two families of tidal deformability parameters:

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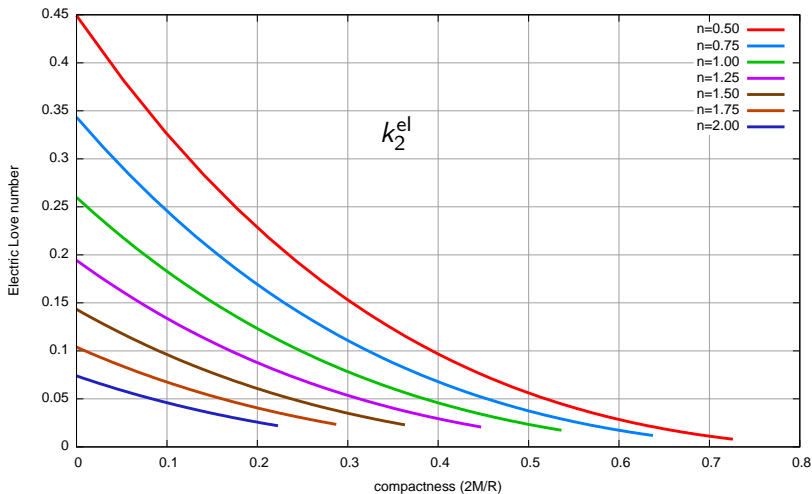
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- Dimensionless tidal Love numbers:

$$k_\ell^{\text{el/mag}} \equiv -\frac{(2\ell-1)!!}{2(\ell-2)!} \frac{\lambda_\ell^{\text{el/mag}}}{R^{2\ell+1}}$$

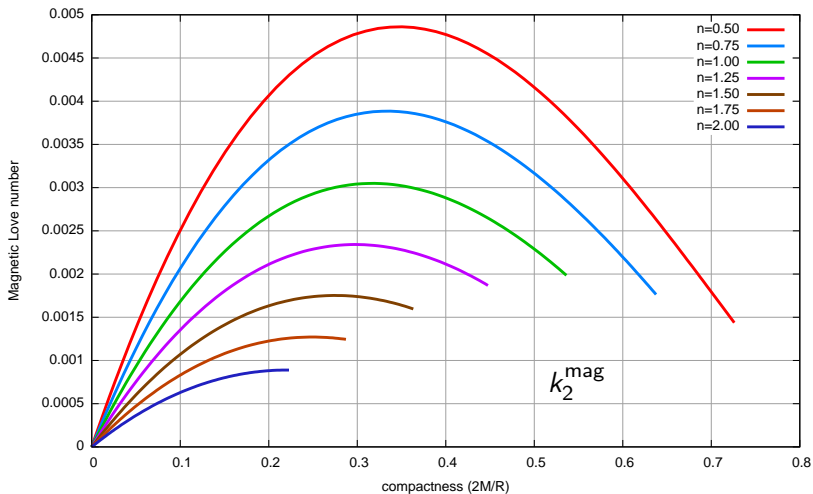
# Love numbers of neutron stars

[Binnington & Poisson, PRD 2009]



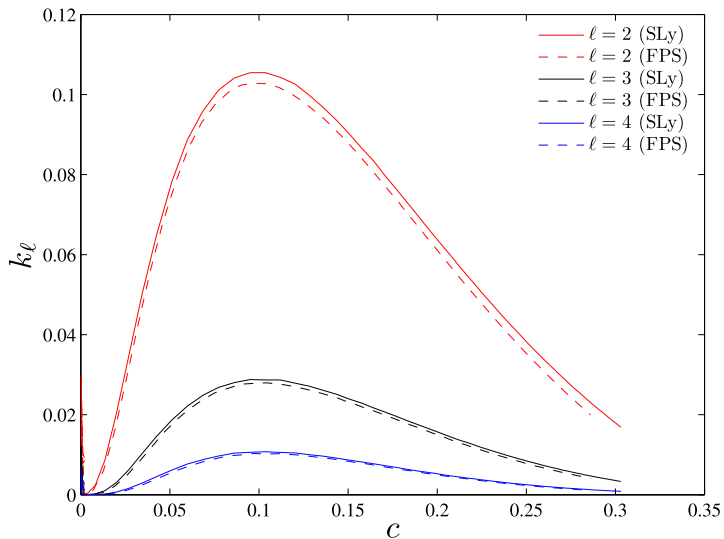
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## Love numbers of *spinning* compact objects

- The spin breaks the spherical symmetry of the background
  - No proportionality between  $(\delta M_L, \delta S_L)$  and  $(\mathcal{E}_L, \mathcal{B}_L)$
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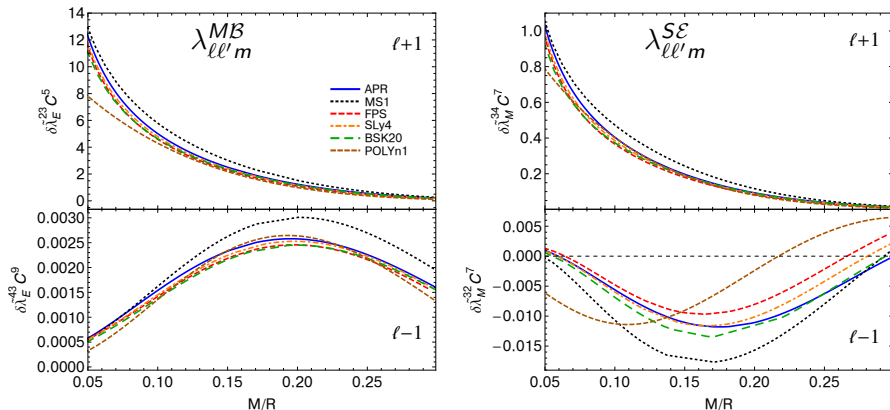
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- Four families of tidal deformability parameters:

$$\begin{aligned} \lambda_{\ell\ell'm}^{M\mathcal{E}} &\equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell'm}} & \lambda_{\ell\ell'm}^{S\mathcal{B}} &\equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell'm}} \\ \lambda_{\ell\ell'm}^{S\mathcal{E}} &\equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{E}_{\ell'm}} & \lambda_{\ell\ell'm}^{M\mathcal{B}} &\equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{B}_{\ell'm}} \end{aligned}$$

# Love numbers of *spinning* neutron stars

[Pani, Gualtieri & Ferrari, PRD 2015]



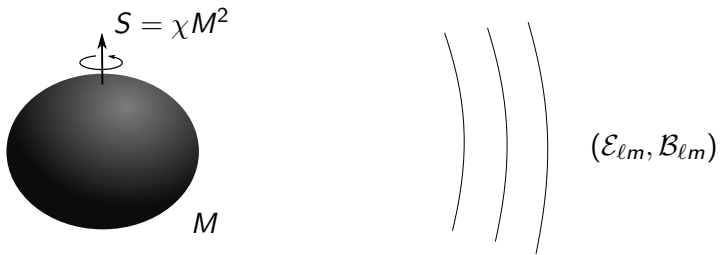
Restricted to an *axisymmetric* tidal perturbation ( $m = 0$ )

## Black holes have *zero* Love numbers

Reference	Background	Tidal field
[Binnington & Poisson 2009]	Schwarzschild	weak, generic $\ell$
[Damour & Nagar 2009]	Schwarzschild	weak, generic $\ell$
[Kol & Smolkin 2012]	Schwarzschild	weak, electric-type
[Chakrabarti et al. 2013]	Schwarzschild	weak, electric, $\ell = 2$
[Gürlebeck 2015]	Schwarzschild	strong, axisymmetric
[Landry & Poisson 2015]	Kerr to $O(S)$	weak, quadrupolar
[Pani et al. 2015]	Kerr to $O(S^2)$	weak, $(\ell, m) = (2, 0)$
[Chia 2020]	Exact Kerr	weak, generic $\ell$

Problem of **fine-tuning** from an Effective-Field-Theory perspective

## Investigating Kerr's Love



$$(\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}) \rightarrow \psi_0 \rightarrow \Psi \rightarrow h_{\alpha\beta} \rightarrow (M_{\ell m}, S_{\ell m}) \rightarrow \lambda_{\ell m}^{M/S, \mathcal{E}/\mathcal{B}}$$

Metric reconstruction through the Hertz potential  $\Psi$

## Perturbed Weyl scalar

- Recall that in the Newtonian limit we established

$$\lim_{c \rightarrow \infty} \psi_0^{\ell m} \propto \mathcal{E}_{\ell m} r^{\ell-2} [1 + 2k_{\ell m} (R/r)^{2\ell+1}] {}_2Y_{\ell m}(\theta, \phi)$$

- For a Kerr black hole the perturbed Weyl scalar reads

$$\psi_0^{\ell m} \propto \left[ \mathcal{E}_{\ell m} + \frac{3i}{\ell+1} \mathcal{B}_{\ell m} \right] R_{\ell m}(r) {}_2Y_{\ell m}(\theta, \phi)$$

- Asymptotic behavior of general solution of static radial Teukolsky equation:

$$R_{\ell m}(r) = \underbrace{r^{\ell-2} (1 + \dots)}_{\text{tidal field } R_{\ell m}^{\text{tidal}}} + \kappa_{\ell m} \underbrace{r^{-\ell-3} (1 + \dots)}_{\text{linear response } R_{\ell m}^{\text{resp}}}$$

## Kerr black hole linear response

$$R_{\ell m}(r) = \underbrace{R_{\ell m}^{\text{tidal}}(r)}_{\sim r^{\ell-2}} + 2k_{\ell m} \underbrace{R_{\ell m}^{\text{resp}}(r)}_{\sim r^{-(\ell+3)}}$$

- The coefficients  $k_{\ell m}$  can be interpreted as the Newtonian Love numbers of a Kerr black hole and read

$$k_{\ell m} = -im\chi \frac{(\ell+2)!(\ell-2)!}{4(2\ell+1)!(2\ell)!} \prod_{n=1}^{\ell} [n^2(1-\chi^2) + m^2\chi^2]$$

- The linear response vanishes identically when:
  - the black hole spin vanishes ( $\chi = 0$ )
  - the tidal field is axisymmetric ( $m = 0$ )
- Reconstruct the Kerr black hole response  $h_{\alpha\beta}^{\text{resp}}$  via  $\Psi^{\text{resp}}$



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- The black hole tidal bulge is **rotated by  $45^\circ$**  with respect to the quadrupolar tidal perturbation
- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} \doteq 0$$

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- For a dimensionless black hole spin  $\chi = 0.1$  this gives

$$|k_{2,\pm 2}| \simeq 2 \times 10^{-3} \quad \longrightarrow \quad \text{black holes are “rigid”}$$

## Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\text{el}} \mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\text{mag}} \mathcal{B}_{ab}$$

- For a **spinning black hole** we have the more general **tensorial** relations

$$\delta M_{ab} = \lambda_{abcd} \mathcal{E}_{cd} \quad \text{and} \quad \delta S_{ab} = \lambda_{abcd} \mathcal{B}_{cd}$$

- To linear order in the black hole spin vector  $S^a$  we find

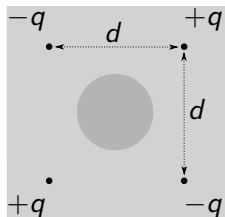
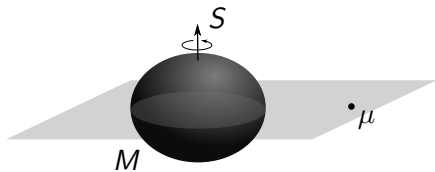
$$\delta M_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{E}^d_{(a} \mathcal{E}_{b)cd}$$
$$\delta S_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{B}^d_{(a} \mathcal{E}_{b)cd}$$

## Love tensor of a Kerr black hole

$$(\lambda_{abcd}) \doteq \frac{\chi}{180} (2M)^5 \begin{pmatrix} \mathbf{l}_{11} & \mathbf{l}_{12} & \mathbf{l}_{13} \\ \mathbf{l}_{12} & -\mathbf{l}_{11} & \mathbf{l}_{23} \\ \mathbf{l}_{13} & \mathbf{l}_{23} & \mathbf{0} \end{pmatrix}$$

$$\mathbf{l}_{11} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{l}_{12} \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\mathbf{l}_{13} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \mathbf{l}_{23} \equiv \begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

## Newtonian static quadrupolar tide



$$\mathcal{E}_{ab} = \frac{\mu}{r^3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\delta M_{ab} \doteq 3Q \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

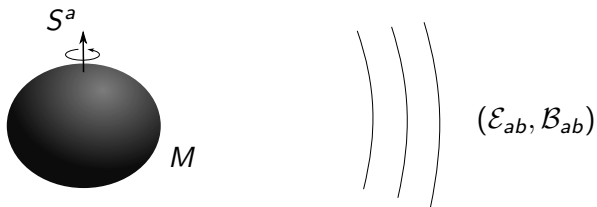
↑

$$\frac{\chi}{180} (2M)^5 \frac{\mu}{r^3} = qd^2$$



# Tidal torquing of a spinning black hole

[Thorne & Hartle 1980; Poisson 2004]



- An arbitrary spinning body interacting with a tidal environment suffers a **tidal torquing**:

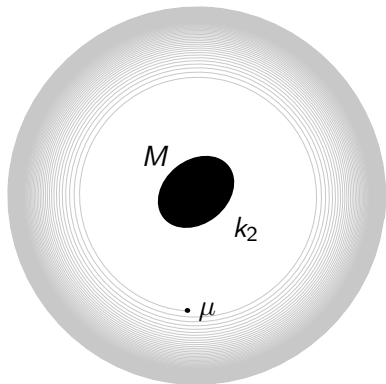
$$\langle \dot{S}^a \rangle = -\epsilon^{abc} \langle M_{bd} \mathcal{E}^d_c + S_{bd} \mathcal{B}^d_c \rangle$$

- Applied to a **spinning black hole** this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi [2\langle E_1 + B_1 \rangle - 3\langle E_2 + B_2 \rangle]$$

# Observing black hole tidal deformability

[Pani & Maselli, IJMPD 2019]



Accumulated **GW phase in LISA**  
band during quasi-circular inspiral  
down to Schwarzschild ISCO:

$$\Phi_{\text{tidal}} \simeq -2 \times 10^3 \left( \frac{10^{-7}}{\mu/M} \right) \left( \frac{k_2}{0.002} \right)$$

↑

like 1st order dissipative self-force

## Summary

- Love numbers of Kerr black holes **do not vanish** in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- **Kerr black holes deform** like any other self-gravitating body, despite being particularly “rigid” compact objects
- This is closely related to the phenomenon of **tidal torquing**
- The black hole tidal deformation contribution to the GW phase of EMRIs could be **detectable by LISA**
- **New black hole test** of the Kerr-like nature of the massive compact objects at the center of galaxies

**Spinning black holes fall in Love!**

## Why analytic continuation?

$$R_{\ell m}(r) = \underbrace{r^{\ell-2} (1 + \dots)}_{\text{tidal field } R_{\ell m}^{\text{tidal}}} + \kappa_{\ell m} \underbrace{r^{-\ell-3} (1 + \dots)}_{\text{linear response } R_{\ell m}^{\text{resp}}}$$

**Ambiguity in the linear response** [Fang & Lovelace 2005; Gralla 2018]

The decaying solution  $R_{\ell m}^{\text{resp}}$  is affected by a radial coord. transfo.

**Ambiguity in the tidal field** [Pani, Gualtieri, Maselli & Ferrari 2015]

The growing solution  $R_{\ell m}^{\text{tidal}} + \alpha R_{\ell m}^{\text{resp}}$  still qualifies as a tidal solution

## Two basis of independent solutions

- Dimensionless radial coordinate and spin parameter:

$$x \equiv \frac{r - r_+}{r_+ - r_-} \quad \text{and} \quad \gamma = \frac{a}{r_+ - r_-}$$

- Smooth and unsmooth solutions:

$$R_{\ell m}^{\text{smooth}} = x^{-2}(1+x)^{-2} \mathbf{F}(-\ell-2, \ell-1, -1+2im\gamma; -x)$$

$$R_{\ell m}^{\text{unsmooth}} = (1+1/x)^{2im\gamma} \mathbf{F}(-\ell+2, \ell+3, 3-2im\gamma; -x)$$

- Tidal and response solutions:

$$R_{\ell m}^{\text{tidal}} = \frac{x^\ell}{(1+x)^2} F(-\ell-2, -\ell-2im\gamma, -2\ell; -1/x) \sim x^{\ell-2}$$

$$R_{\ell m}^{\text{resp}} = \frac{x^{-\ell-1}}{(1+x)^2} F(\ell-1, \ell+1-2im\gamma, 2\ell+2; -1/x) \sim x^{-\ell-3}$$