Physique et géométrie des trous noirs

Alexandre Le Tiec

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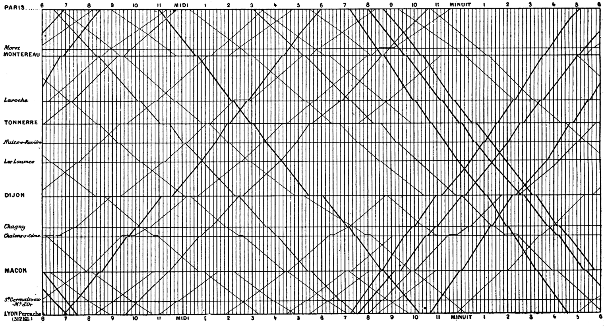
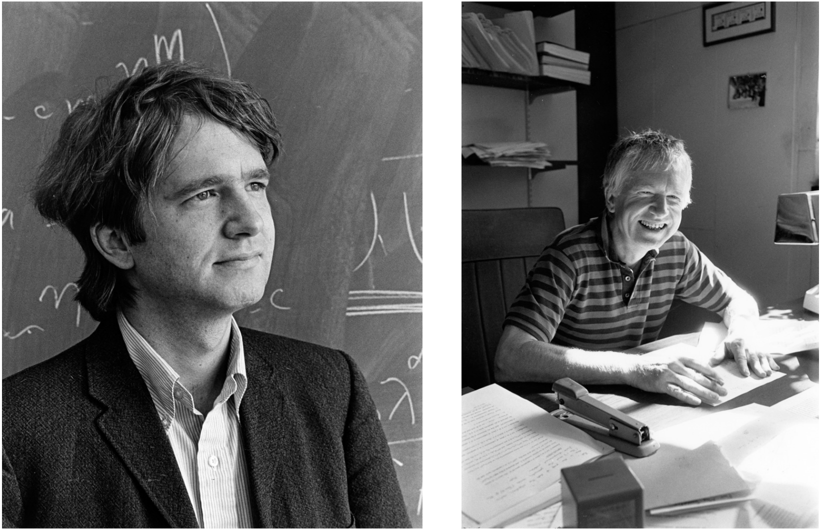
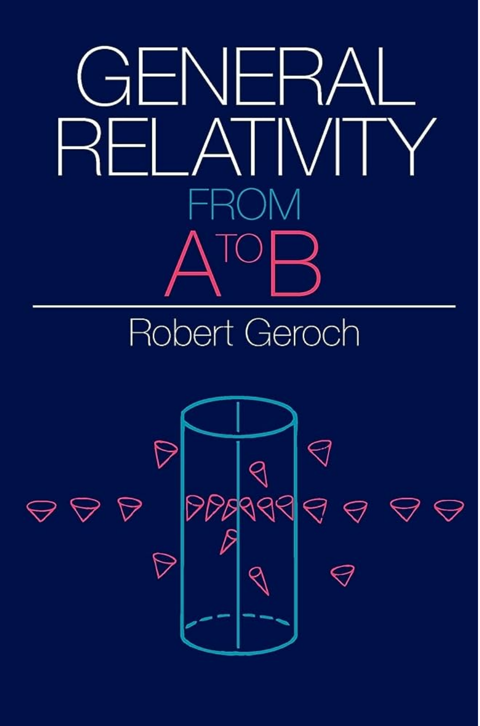


Fig. 7. Graphique de la marche des trains sur un chemin de fer, d'après la méthode de Ibry.

Robert Geroch (1942–)





GENERAL RELATIVITY FROM A TO B

Robert Geroch

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B GENERAL RELATIVITY

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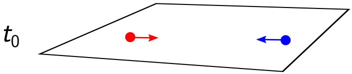
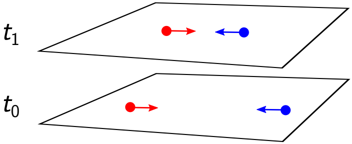
3. Difficultés galiléennes
4. Espace-temps relativiste
5. Gravitation relativiste
6. Trous noirs

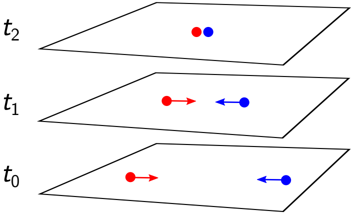
Espace, temps et espace-temps



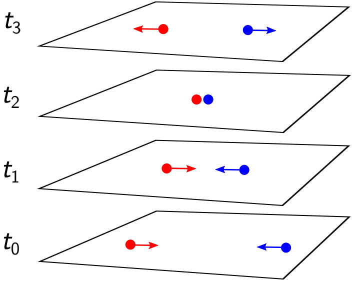
Espace, temps et espace-temps



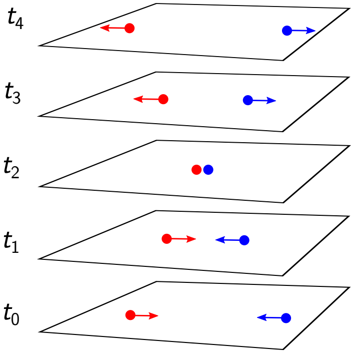
Espace, temps et espace-temps



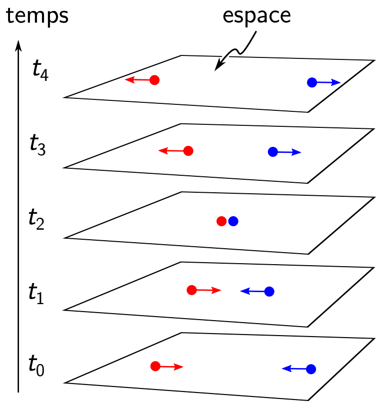
Espace, temps et espace-temps



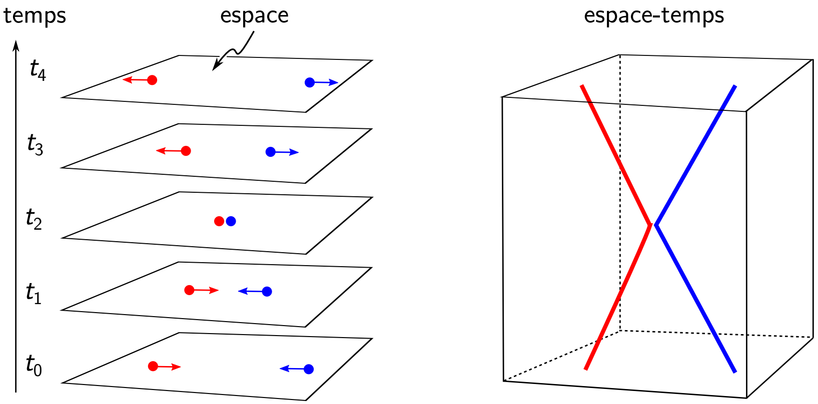
Espace, temps et espace-temps



Espace, temps et espace-temps



Espace, temps et espace-temps



Plan de l'exposé

1. Espace-temps aristotélicien
2. Espace-temps galiléen
3. Difficultés galiléennes
4. Espace-temps relativiste
5. Gravitation relativiste
6. Trous noirs

Repérer un événement dans l'espace-temps

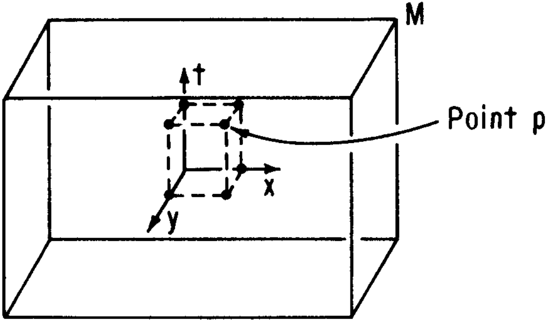


Fig. 3
The location of a point *p* in space-time by its Aristotelian coordinates.

La ligne d'univers d'un individu statique

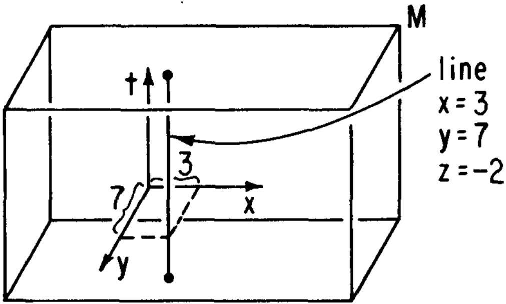


Fig. 4
The world-line of the individual whose badge reads $x = 3$, $y = 7$, $z = -2$ is the vertical straight line with these coordinates.

Espace absolu et temps absolu

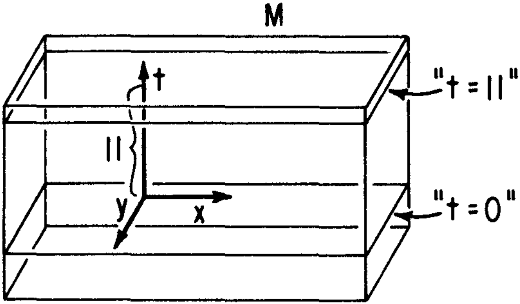


Fig. 5
The horizontal 3-plane given by $t = 11$ in space-time represents "all space at time $t = 11$."

Distance spatiale et intervalle de temps

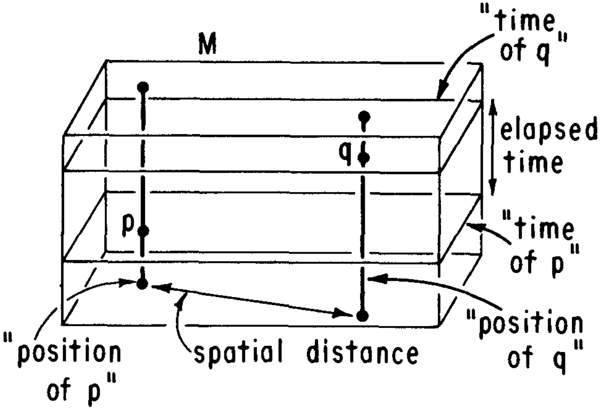


Fig. 6
Points p and q of space-time are described, in the Aristotelian view, by their positions in space (vertical lines) and their times of occurrence (horizontal 3-planes). The spatial distance and elapsed time between the events is then given, respectively, by the distance between the vertical lines and the distance between the horizontal 3-planes.

La “dynamique” d’une particule dans l’espace-temps

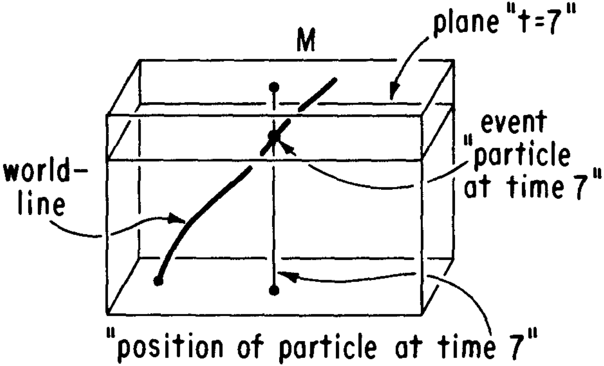


Fig. 7
The dynamics, according to the Aristotelian view, of a particle in space-time.

Exemples de particules dans l'espace-temps

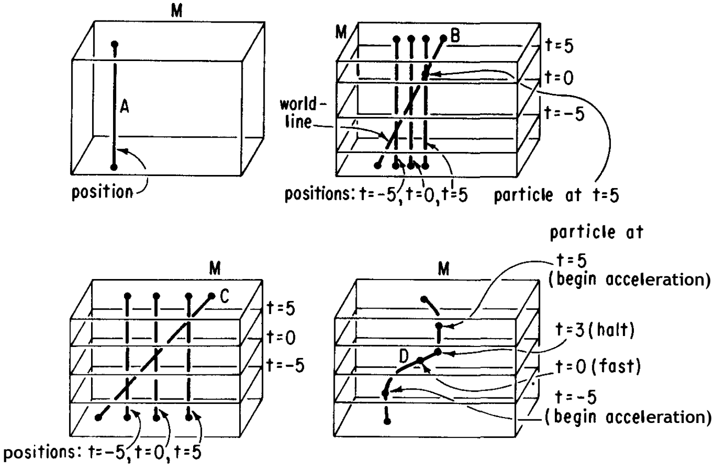


Fig. 8
Four examples of the interpretation of the space-time diagram of a particle. Particle *A* is at rest. Particle *B* is moving at a small, constant velocity, and particle *C* at a larger, constant velocity. Particle *D* is executing a more complicated motion.

La collision entre deux particules

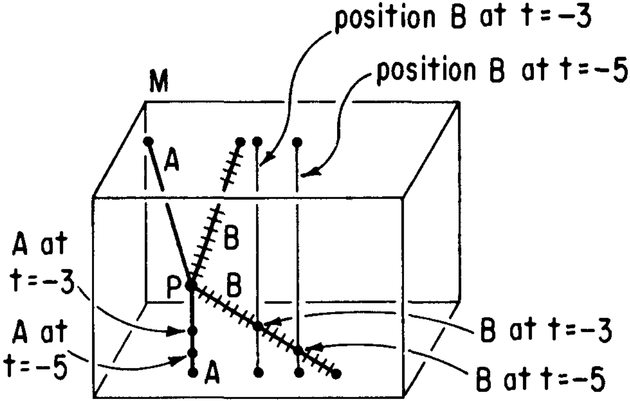


Fig. 9
The space-time diagram of the collision between two particles.

La surface d'univers d'une corde

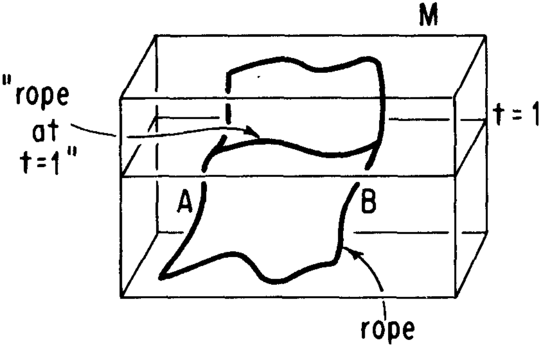


Fig. 10
The space-time diagram of a rope is a two-dimensional surface.

La surface d'univers d'une corde

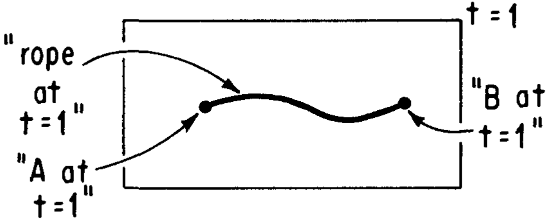


Fig. 11
To obtain the spatial configuration of the rope of figure 10 at a given time, one intersects the world-surface of the rope with the horizontal 3-plane representing that time. Here, then, is that spatial configuration at time $t = 1$.

Exemples de cordes dans l'espace-temps

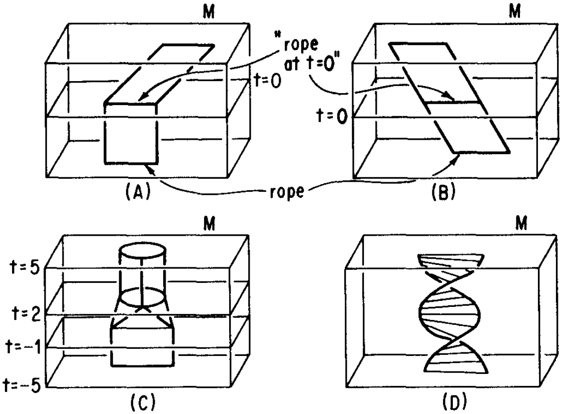


Fig. 12

Four examples of the interpretation, in the Aristotelian view, of the world-surface of a rope. The rope in (*A*) is initially at rest, but then begins to move in the direction of its extension. The rope in (*B*) moves at constant velocity orthogonally to the direction of its extension. The rope in (*C*) forms itself into a circle. Finally, the rope in (*D*) is rotating about its midpoint.

Exemples de planètes dans l'espace-temps

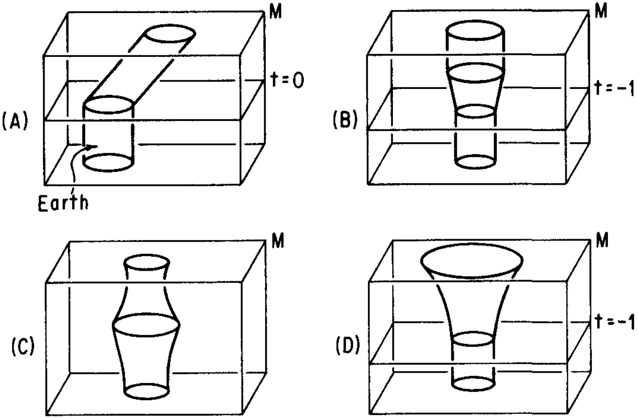


Fig. 13
Four examples of the interpretation, in the Aristotelian view, of the world-region of a planet.

Diagrammes d'espace-temps plus compliqués

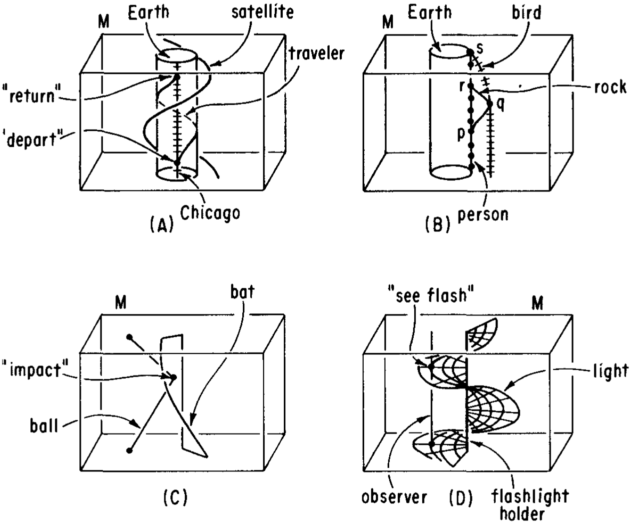


Fig. 14
Four examples of the interpretation, in the Aristotelian view, of more complicated space-time diagrams.

Émission d'un photon ou d'un faisceau lumineux

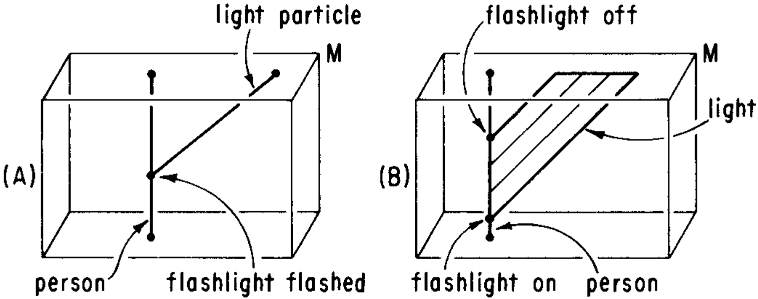


Fig. 15
The space-time diagrams representing (*A*) the emission of a momentary flash of light in a given direction, and (*B*) the emission of a beam of light in a given direction.

Le cône de lumière d’un événement

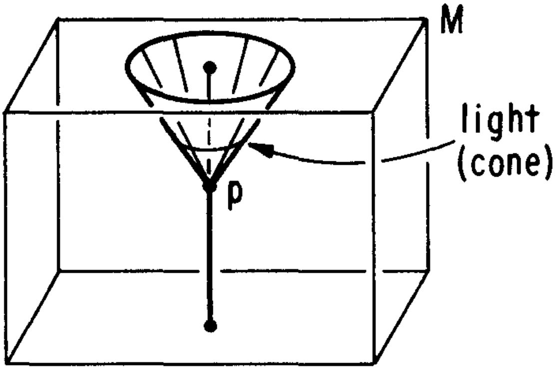
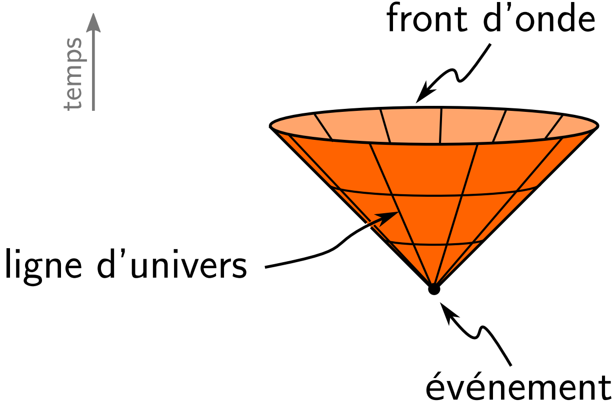


Fig. 16
The light-cone of event *p* is the locus of all events illuminated by a flash of light emitted in all directions from *p*.

Le cône de lumière d'un événement



Plan de l'exposé

Notre perspective

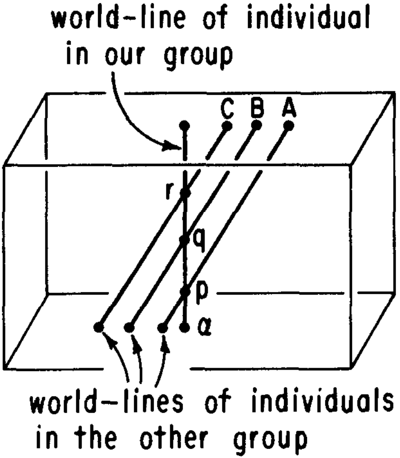


Fig. 17
The description, within our Aristotelian view, of a second Aristotelian setup. The world-lines of the members of the other group are straight and parallel, but are not, in general, vertical.

Leur perspective

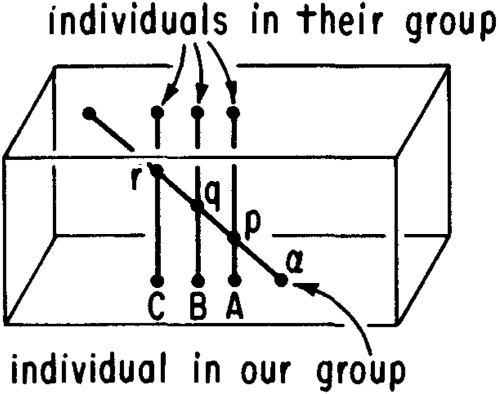


Fig. 18
Figure 17 as drawn by the other Aristotelian group.

Notre perspective et leur perspective

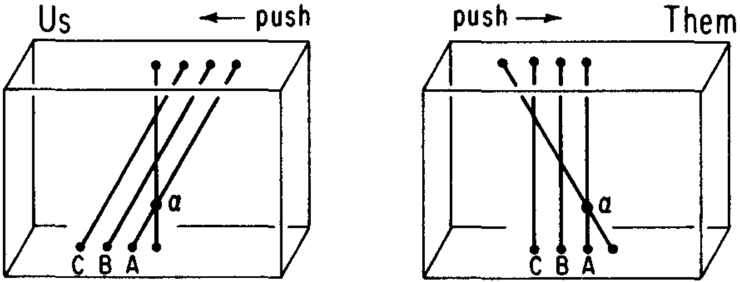


Fig. 19
The geometrical relationship between figures 17 and 18. The two figures are related to each other by sliding horizontal 3-planes over each other.

Notre perspective et leur perspective

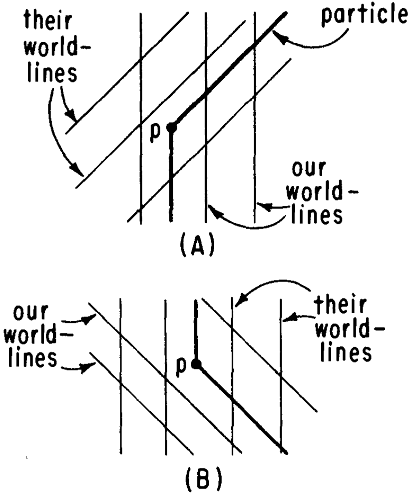


Fig. 20
Space-time diagrams for a particle which initially remains with a member of our group but, at event *p*, joins their group. Figure (*A*) is as would be drawn by our Aristotelian setup, (*B*) as by theirs.

Notre perspective et leur perspective

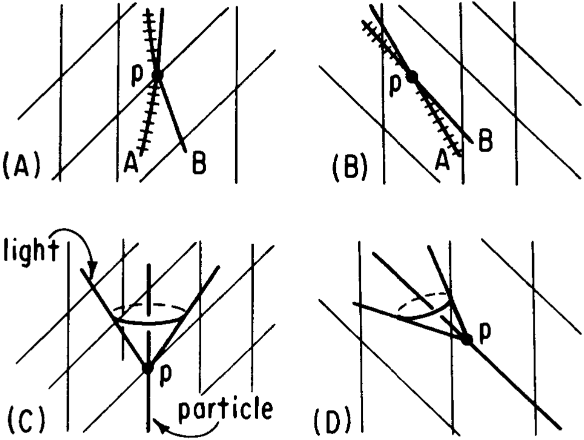


Fig. 21
Two examples of translating space-time diagrams from one Aristotelian setup to another. Figures (*A*) and (*B*) show the collision of two particles; (*C*) and (*D*) the emission of a flash of light in all directions.

Principe de relativité galiléenne

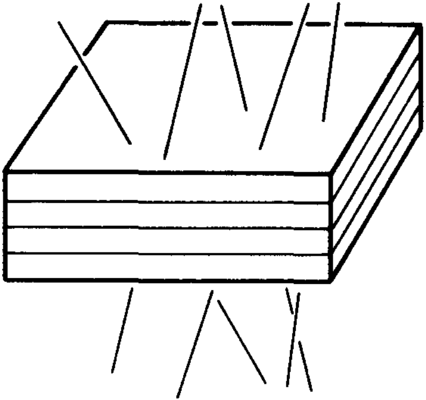


Fig. 22

Space-time, according to the Galilean view. The straight lines represent world-lines of particles, all moving at constant relative velocities. No one family of lines is preferred.

Notion de distance spatiale entre deux événements

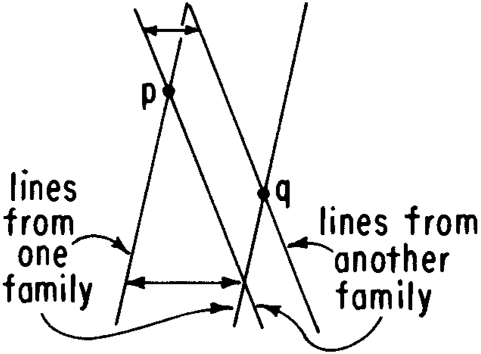


Fig. 23
"Spatial distance between two events" does not make sense in the Galilean view. The value one obtains for the "spatial distance" will in general depend on which Aristotelian group is determining it.

Notion de distance spatiale entre deux événements

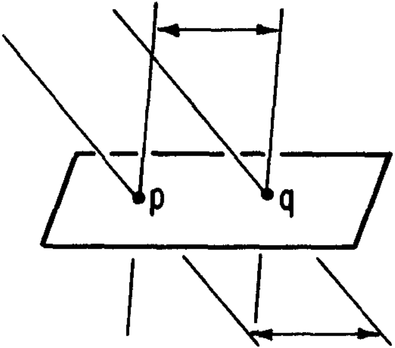


Fig. 24
"Spatial distance" does make sense, even in the Galilean view, for two events which occur at the same time.

Plan de l'exposé

Une expérience de pensée

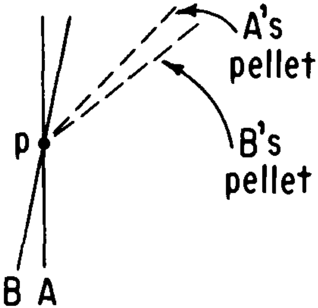


Fig. 25
Space-time diagram for a pellet-shooting experiment. Individuals *A* and *B* meet at event *p*, at which event each shoots a small pellet, using identical spring guns.

Une expérience de pensée

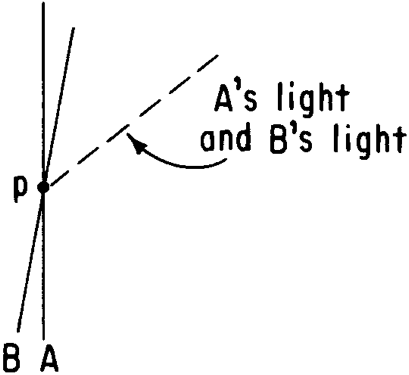
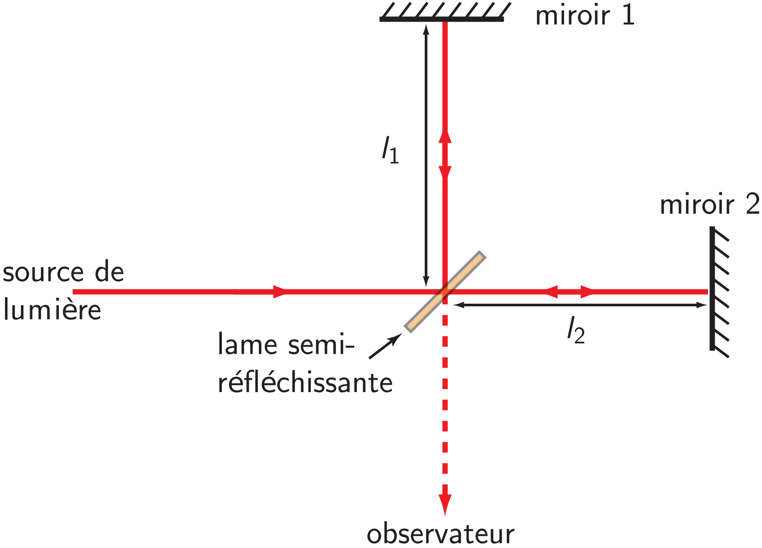
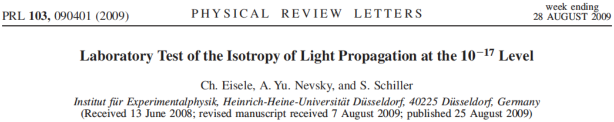


Fig. 26
Figure 25, but now with pulses of light replacing the pellets.

Expériences de Michelson et Morley



Isotropie de la vitesse de propagation de la lumière



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Laboratory Test of the Isotropy of Light Propagation at the 10^{-17} Level

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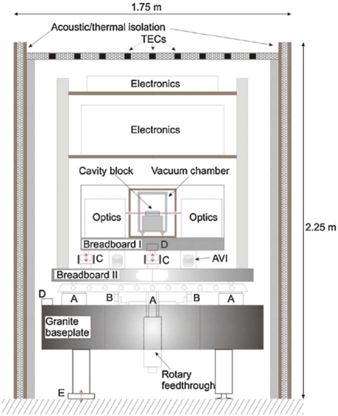
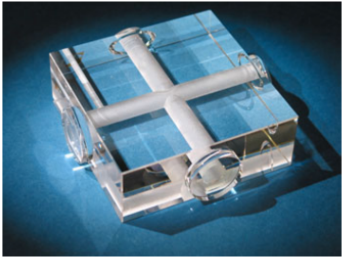
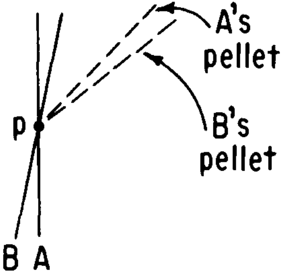
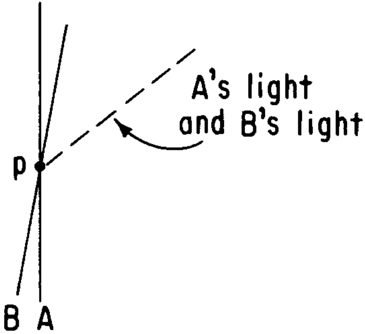




précision relative $\sim 10^{-17}$

Loi de composition des vitesses relativiste





$$w = \frac{u+v}{1+\frac{uv}{c^2}}$$

$$u = c \quad \Longrightarrow \quad w = c$$

Postulats de la théorie de la relativité restreinte

Postulat de relativité

Les équations décrivant les lois de la physique prennent la même forme dans tous les référentiels inertiels/galiléens

“Le mouvement est comme rien.”

Postulats de la théorie de la relativité restreinte

Postulat de relativité

Les équations décrivant les lois de la physique prennent la même forme dans tous les référentiels inertiels/galiléens

"Le mouvement est comme rien."

Postulat d'invariance de la célérité

Dans le vide, la lumière se propage à une vitesse constante c, indépendamment de l'état de mouvement de la source

Postulats de la théorie de la relativité restreinte

Postulat de relativité

Les équations décrivant les lois de la physique prennent la même forme dans tous les référentiels inertiels/galiléens

“Le mouvement est comme rien.”

Postulat d’invariance de la célérité

Dans le vide, la lumière se propage à une vitesse constante c, indépendamment de l’état de mouvement de la source

→ à l’origine du caractère **contre-intuitif** de la relativité

Libre choix du système d'unités

- Par *définition* du mètre, la vitesse de propagation de la lumière dans la vide est

$$c = 2.99792458 \times 10^8 \text{ m} \cdot \text{s}^{-1}$$

- Alternativement, nous pouvons choisir comme unité de distance la *seconde-lumière*, telle que

$$\boxed{c = 1}$$

Désintégration des muons atmosphériques

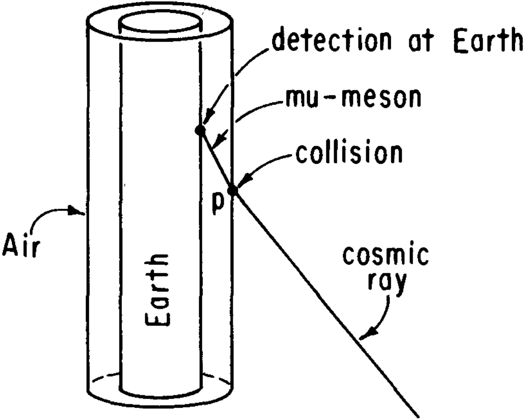


Fig. 29

Space-time diagram for a mu-meson experiment. A cosmic ray collides with the atmosphere at event p, releasing a mu-meson, which is finally detected at the surface of the earth.

Désintégration des muons atmosphériques

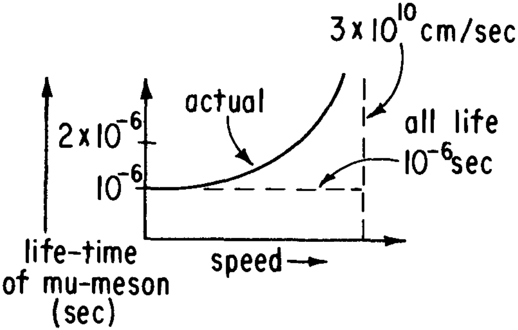
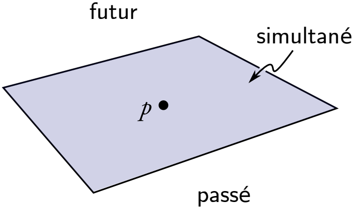


Fig. 30
Graph of the lifetime of a mu-meson as a function of its speed.

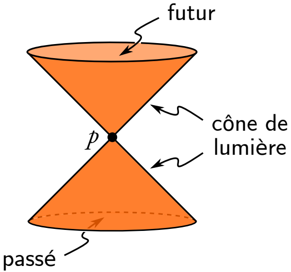
$$\Delta\tau = \Delta t\,\sqrt{1 - v^2}$$

Plan de l'exposé

Causalité

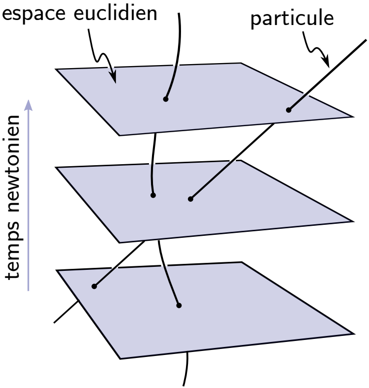


Physique pré-relativiste

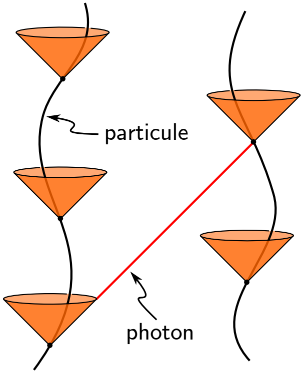


Relativité restreinte

Causalité



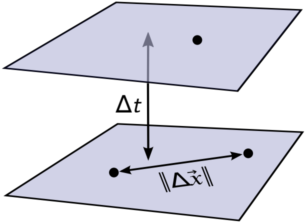
Physique pré-relativiste



Relativité restreinte

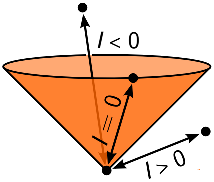
Invariants

$$\Delta t,\ \|\Delta \vec{x}\|$$



Physique pré-relativiste

$$I \equiv -(\Delta t)^2 + \|\Delta \vec{x}\|^2$$



Relativité restreinte

Invariants

$$\Delta t,\ \|\Delta\vec{x}\|$$

Groupe de Galilée

- Rotations spatiales
- Translations spatiales
- Translations temporelles
- Transformations de Galilée

$$\begin{aligned} t' &= t \\ \vec{x}' &= \vec{x} - \vec{v}t \end{aligned}$$

$$I \equiv -(\Delta t)^2 + \|\Delta\vec{x}\|^2$$

Groupe de Poincaré

- Rotations spatiales
- Translations spatiales
- Translations temporelles
- Transformations de Lorentz

$$\begin{aligned} t' &= \gamma\left(t - \vec{v}\cdot\vec{x}_{\|}\right) \\ \vec{x}'_{\|} &= \gamma\left(\vec{x}_{\|} - \vec{v}t\right) \end{aligned}$$

Invariants

$$\Delta t,\ \|\Delta\vec{x}\|$$

Groupe de Galilée

- Rotations spatiales
- Translations spatiales
- Translations temporelles
- Transformations de Galilée

$$\begin{aligned} t' &= t \\ \vec{x}' &= \vec{x} - \vec{v}t \end{aligned}$$

$$I \equiv -(\Delta t)^2 + \|\Delta\vec{x}\|^2$$

Groupe de Poincaré

- Rotations spatiales
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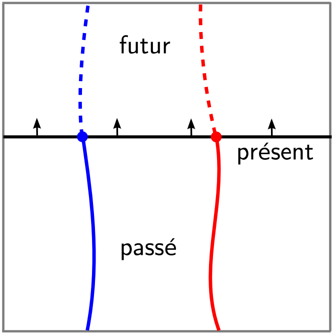
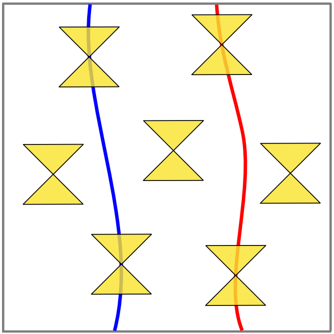
- Translations temporelles
- Transformations de Lorentz

$$\begin{aligned} t' &= \gamma\left(t - \vec{v}\cdot\vec{x}_{\|}\right) \\ \vec{x}'_{\|} &= \underset{\substack{\uparrow \\ 1/\sqrt{1-v^2}}}{\gamma}\left(\vec{x}_{\|} - \vec{v}t\right) \end{aligned}$$

Disparition du présent



Physique pré-relativiste



Relativité restreinte

Représentation d’une horloge dans l’espace-temps

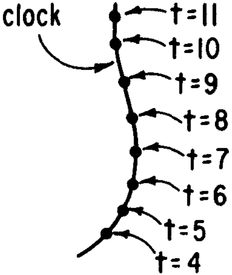


Fig. 33

Representation of a clock in space-time. The clock possesses a world-line, and furthermore assigns a number *t*, the time-reading of the clock, to each event on that world-line.

Échange d'informations par signaux lumineux

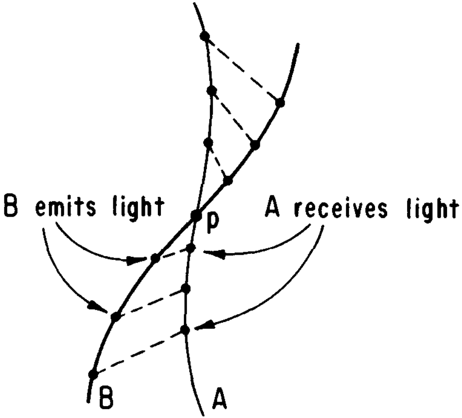
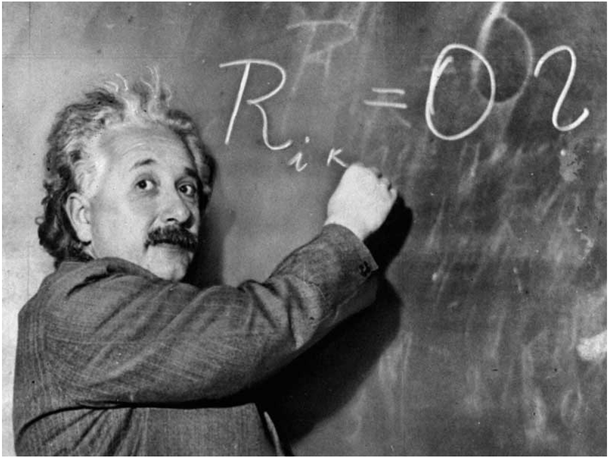


Fig. 61

Two observers, *A* and *B*, meet at event *p*. Observer *A* receives light from *B*, by means of which *B*'s clock-readings are transmitted to *A*.

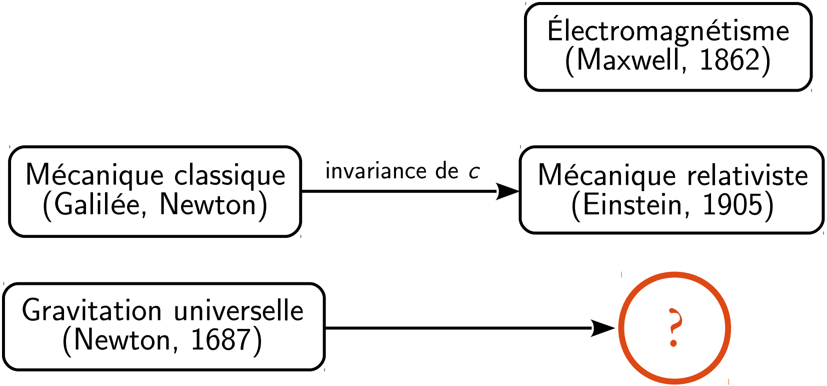
Plan de l'exposé

1. Espace-temps aristotélicien
2. Espace-temps galiléen
3. Difficultés galiléennes
4. Espace-temps relativiste
5. Gravitation relativiste
6. Trous noirs



La *relativité générale* est la théorie de l'**espace**, du **temps** et de la **gravitation** formulée par Albert Einstein en 1915

Relativité et gravitation



Électromagnétisme
(Maxwell, 1862)

Mécanique classique
(Galilée, Newton)

invariance de c

Mécanique relativiste
(Einstein, 1905)

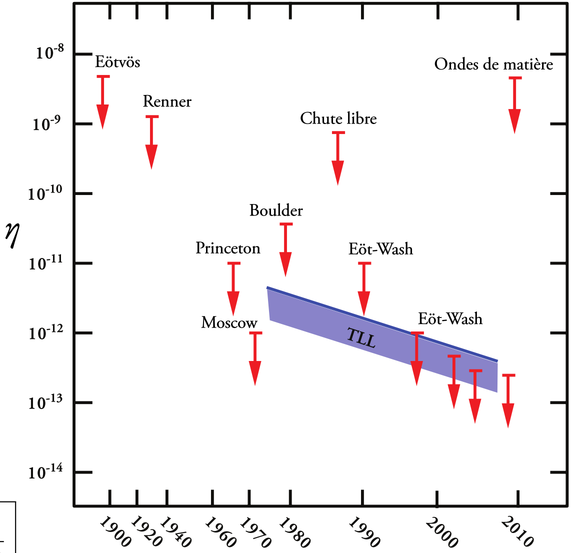
Gravitation universelle
(Newton, 1687)

?

Physique pré-relativiste

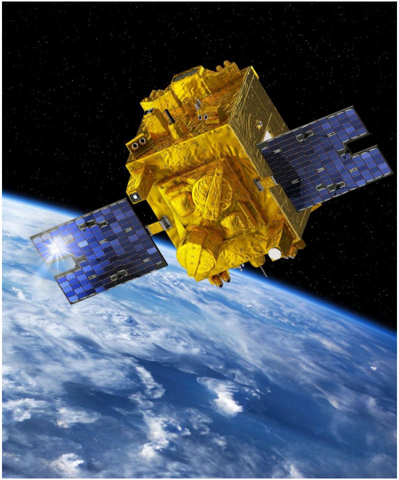
Relativité restreinte

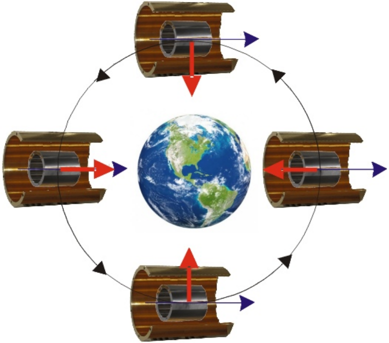
Universalité de la chute libre



$$\boxed{\eta \equiv \frac{|a_1 - a_2|}{\frac{1}{2}(a_1 + a_2)}}$$

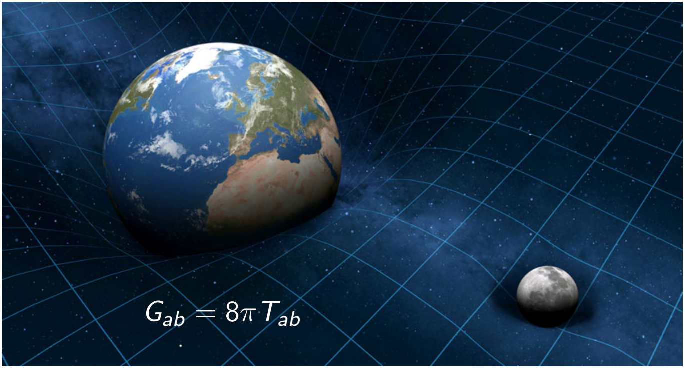
La mission MICROSCOPE





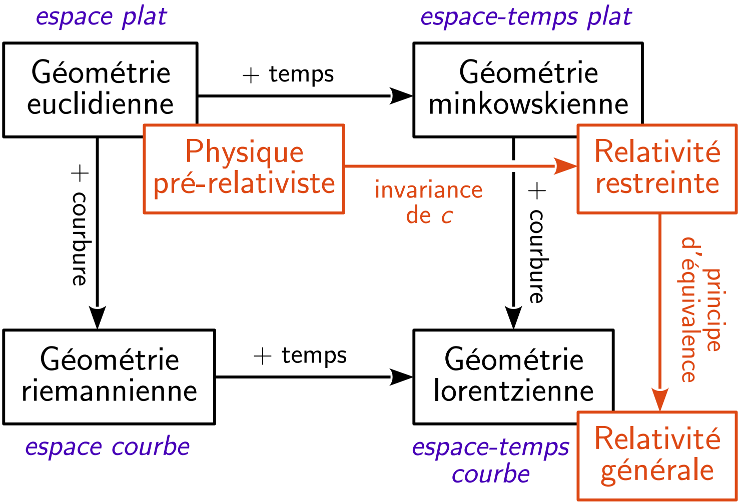
$$\boxed{\eta < 10^{-15}}$$

L'espace-temps est courbe

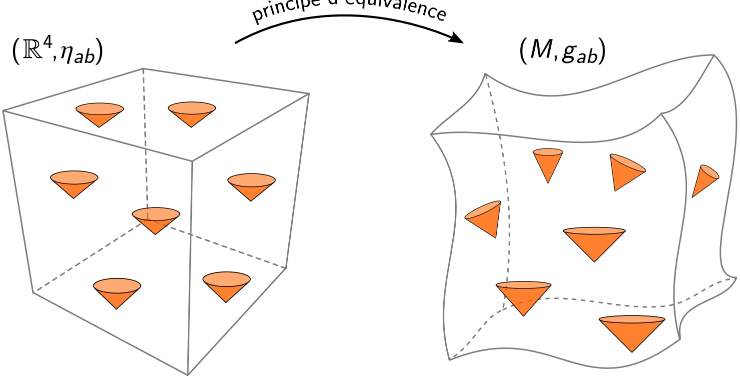


La gravitation est la manifestation de la **courbure de l'espace-temps** par la masse et l'énergie de la matière

Gravitation et géométrie



Relativité restreinte → Relativité générale



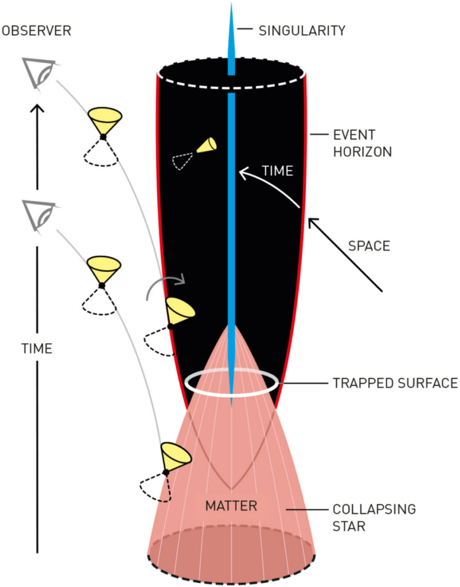
Relativité restreinte (1905)

Relativité générale (1915)

Plan de l'exposé

1. Espace-temps aristotélicien
2. Espace-temps galiléen
3. Difficultés galiléennes
4. Espace-temps relativiste
5. Gravitation relativiste
6. **Trous noirs**

L'effondrement gravitationnel d'une étoile



L'espace-temps d'un trou noir

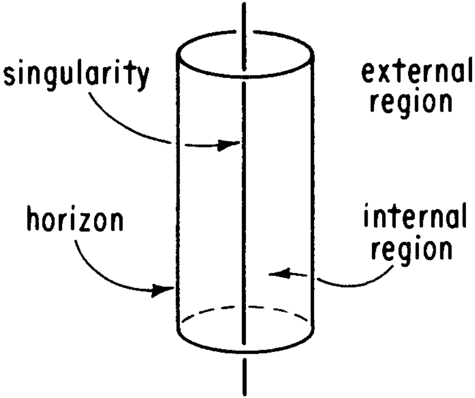


Fig. 80

The set of events for a black-hole space-time. We divide the events into three classes: those in the external region, on the horizon, and in the internal region.

Structure des cônes de lumière

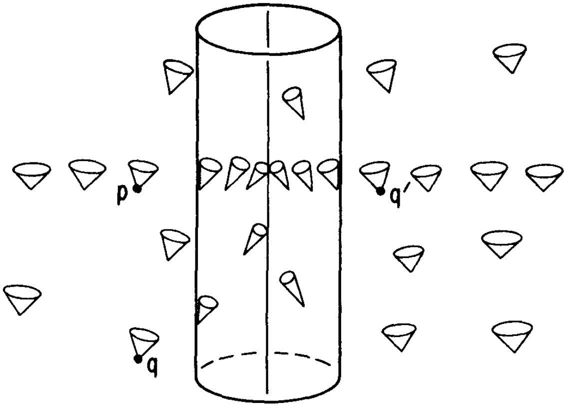


Fig. 81

The light-cones for the black-hole space-time. The cones are "vertical" in the distant external region, become tangent to the horizon, and then, in the internal region, lean toward the singularity. It is intended that the figure have the symmetries of translation up and down, and rotation about the axis.

Structure des cônes de lumière

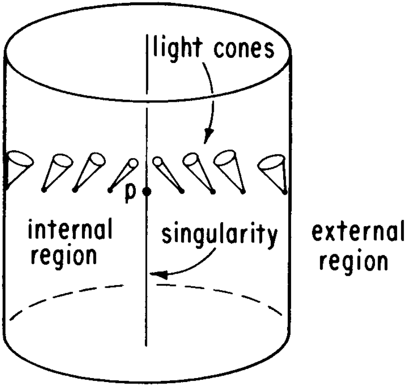


Fig. 82
More detailed view of the light-cones near the singularity.

Trois observateurs proches d'un trou noir

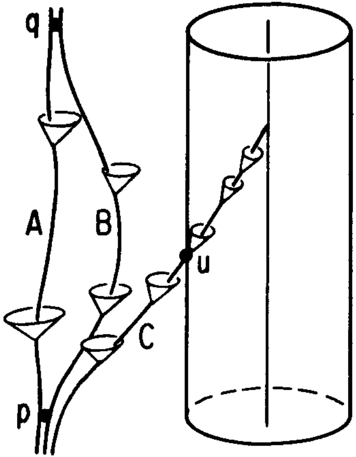


Fig. 83

Three observers in our black-hole space-time. Observer *A* remains in the distant external region, *B* ventures somewhat closer to the hole, and *C* passes through the horizon to the interior of the hole.

Trois destins possibles pour l'observateur C

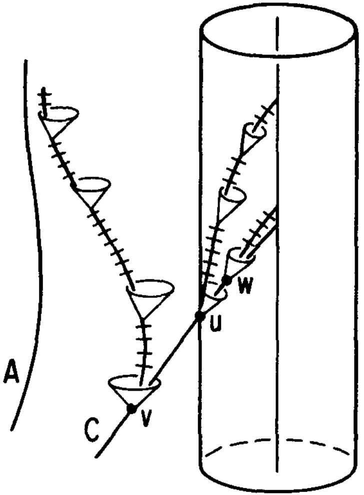


Fig. 84
The result of *C*'s deciding, at various events along his world-line, that he wishes to return to the external region.

Comment manquer un rendez-vous chez le dentiste

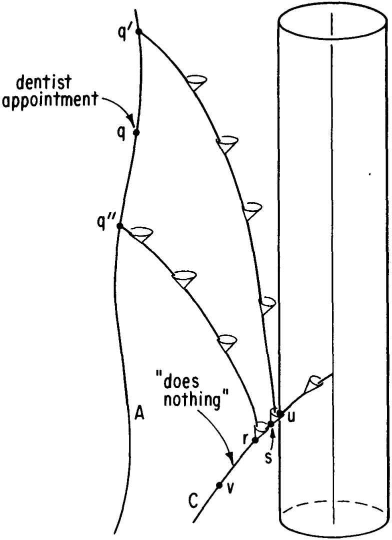
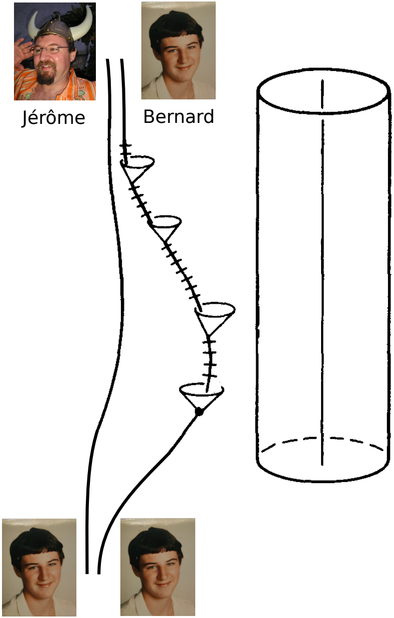


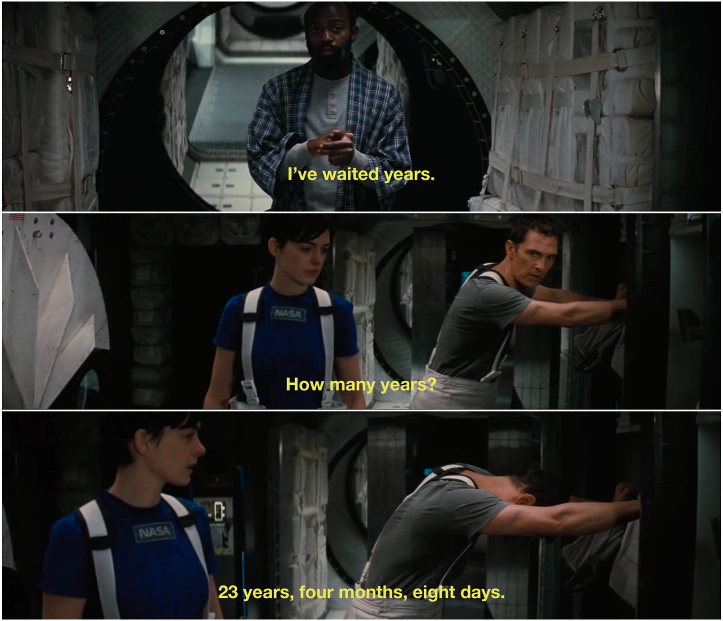
Fig. 85
The nearer to event u that C arrives at his decision to return to the external region, the later C arrives in this region.

Comment voyager dans le futur



Comment voyager dans le futur

Comment voyager dans le futur



Comment manquer ses adieux

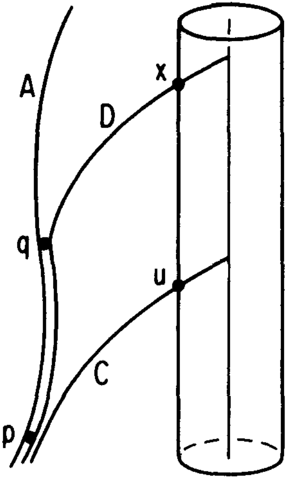


Fig. 86
A space-time diagram representing two individuals, *C* and *D*, who enter the black hole, *D* later than *C*.

Les retrouvailles de C et D sont possibles

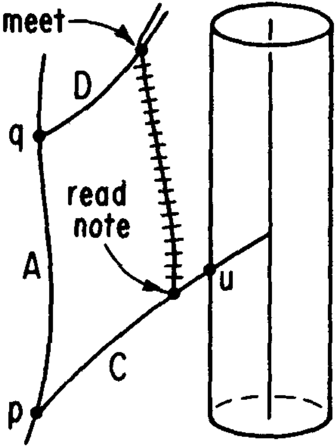


Fig. 87
The result of *C*'s deciding, before event *u*, to return to ***D***.

Les retrouvailles de C et D sont encore possibles

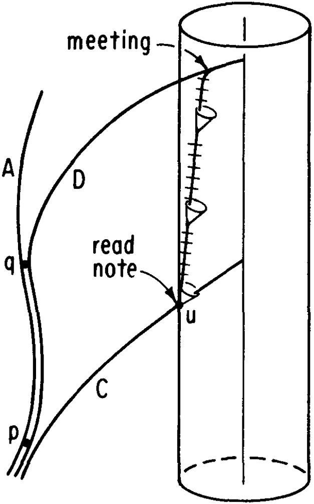


Fig. 88
The result of *C*'s deciding, at event *u*, to return to *D*. Now, *C* and *D* are only able to meet within the internal region.

Les retrouvailles de C et D ne sont plus possibles

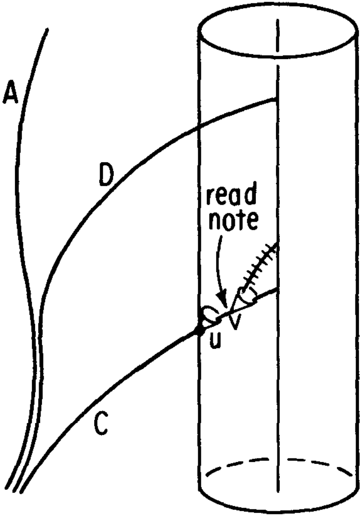


Fig. 89
The result of *C*'s deciding, after event *u*, to return to *D*. In this case, *C* is unable to do so.

L’observateur A émet des rayons lumineux

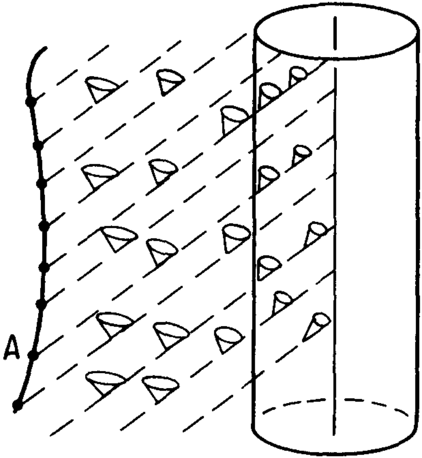


Fig. 90
Space-time diagram representing observer *A*’s emission of light, which enters the black hole.

L’observateur C observe l’observateur A

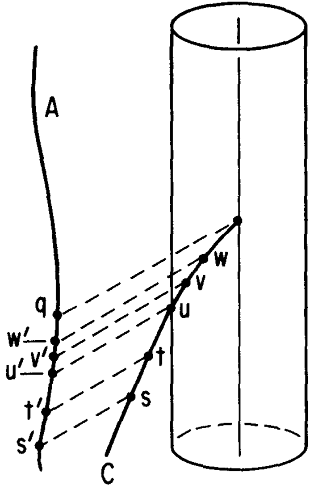


Fig. 91
Light from observer *A* as seen by observer *C*, who enters the black hole. *C* is able to maintain surveillance of *A* during his journey.

L’observateur C observe l’observateur A

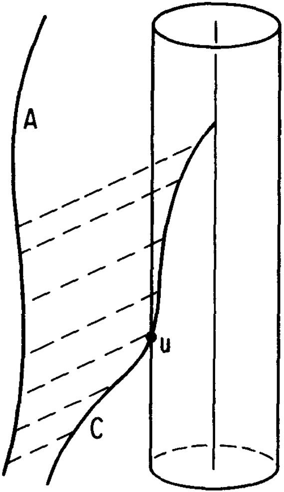


Fig. 92
By accelerating away from the singularity just after event *u*, *C* is able to watch *A*’s activities for a longer time.

L'observateur C émet des rayons lumineux

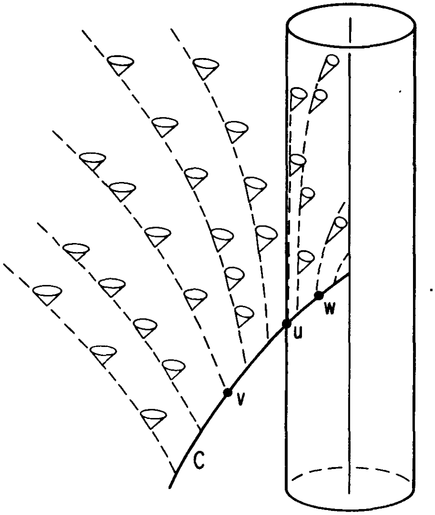


Fig. 93
Light-rays as emitted by an observer *C* who enters the black hole.

L'observateur A observe l'observateur C

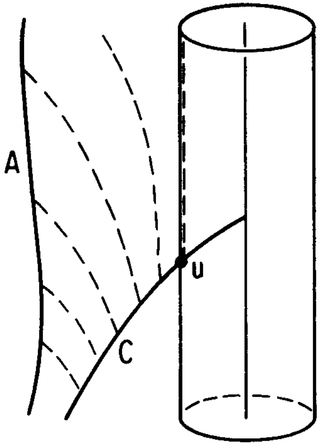


Fig. 94
A watches *C* by means of the light received from *C*. In this case, *A* remains outside the hole, while *C* goes in. The result is that *A*, at any of his moments, can still see *C*, but that *C*'s activities after entering the hole are forever hidden from *A*.

L'observateur D observe l'observateur C

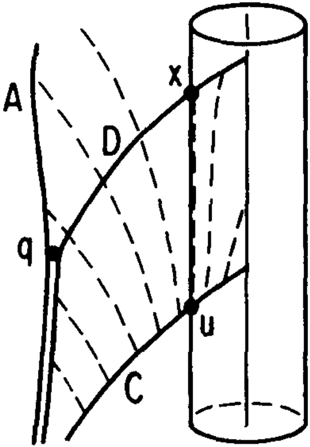


Fig. 95

Observer *D*, by deciding to enter the black hole, is even able to watch *C*'s entry into the black hole.

Les impressions visuelles de l'observateur A

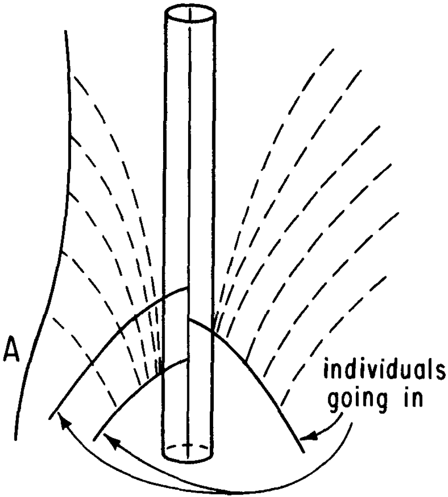


Fig. 96
A's visual impressions on looking at the black hole from the external region. *A* is able, at all times, to see every individual who has ever entered the hole.

L'observateur A "assure" l'observateur C

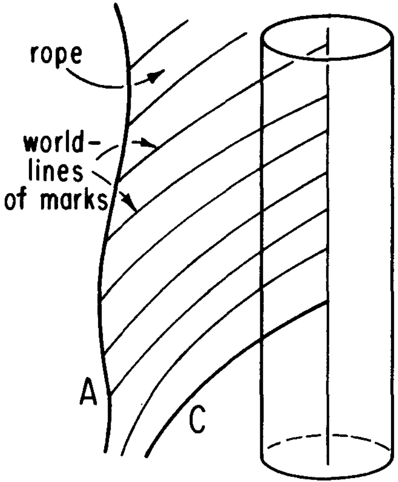


Fig. 97

Observer *A* lowers *C* into the hole by means of a rope, and continues to feed out the rope at all later times. Included in the figure are the world-lines of "marks" made on the rope. The rope, then, is continually being fed into the singularity.

L'observateur A essaie de remonter l'observateur C

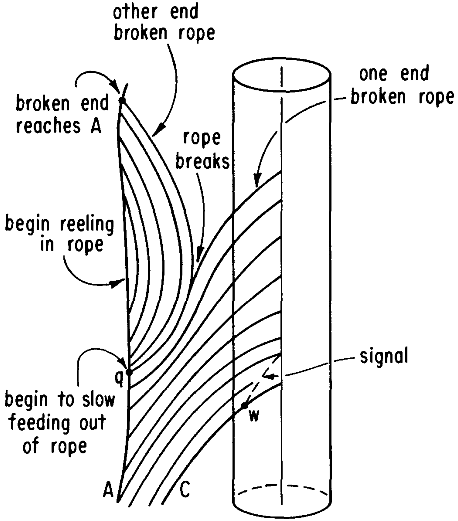


Fig. 98
A space-time diagram of what happens if *A* decides to retrieve *C* by pulling on the rope. Rather than *C*'s returning, the rope breaks.

Pour en savoir plus

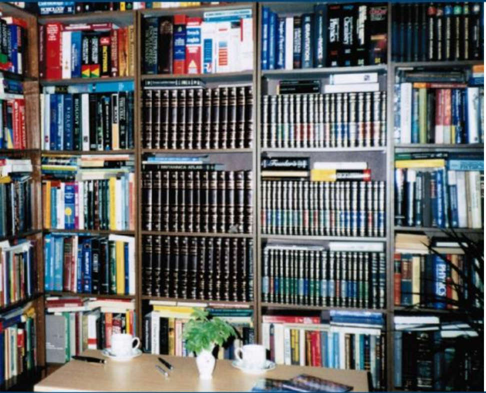
Ouvrages de vulgarisation

- J.-P. Luminet, *Les trous noirs en 100 questions*, Tallandier, 2022
- A. Riazuelo, *Les trous noirs*, Vuibert, 2016
- L. Susskind, *Trous noirs*, Folio, 2012
- J.-P. Lasota, *La science des trous noirs*, Odile Jacob, 2010
- T. Damour, *Si Einstein m'était conté*, Cherche Midi, 2005
- K. Thorne, *Trous noirs et distorsions du temps*, Flammarion, 1997

Ouvrages techniques

- N. Deruelle & J.-P. Uzan, *Théories de la relativité*, Belin, 2014
- D. Langlois, *Introduction à la relativité*, Vuibert, 2011
- E. Gourgoulhon, *Relativité restreinte*, EDP Sciences, 2010
- J.-P. Pérez, *Relativité et invariance*, Dunod, 2005

Lecture Notes Series



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Although the book contains lecture notes written in 1972, it is (and will remain) an excellent introduction to general relativity, which covers its physical foundations, its mathematical formalism, the classical tests of its predictions, its application to cosmology, a number of specific and important issues (such as the initial value formulation of general relativity, signal propagation, time orientation, causality violation, singularity theorems, conformal transformations, and asymptotic structure of spacetime), and the early approaches to quantization of the gravitational field.

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General Relativity

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