Tidal Love numbers of Kerr black holes clarified

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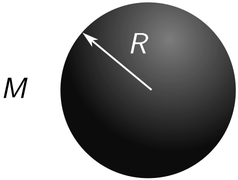
Phys. Rev. Lett. **126** (2021) 131102
Phys. Rev. D **103** (2021) 084021

What's new since last Capra?

- Analytic continuation in multipolar order ℓ vindicated
  [Page 1976; Chia 2020; Charalambous et al. 2021]
- Connection to Kerr black hole tidal torquing
  [Thorne & Hartle 1980; Poisson 2004]
- Purely dissipative nature of Kerr black hole tidal deformability
  [Chia 2020; Goldberger et al. 2020; Charalambous et al. 2021]
- Do Kerr black hole tidal Love numbers vanish, yes or no?!

Agreement on maths but disagreement on **nomenclature**

Newtonian theory of Love numbers



$$U = \frac{M}{r}$$

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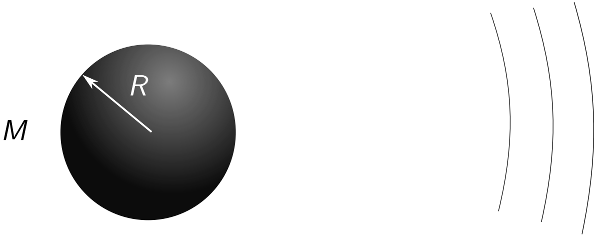

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\rm ext}(\mathbf{0})$$

$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab}$$

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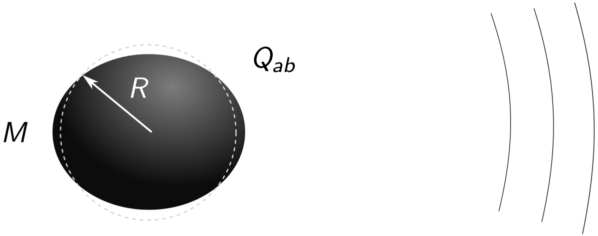

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(\mathbf{0})$$

$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab} + \frac{3}{2} \frac{x^a x^b Q_{ab}}{r^5}$$

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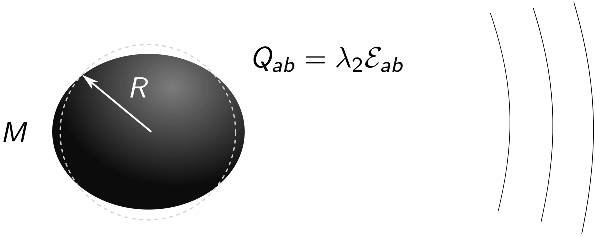

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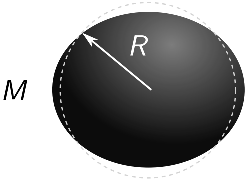

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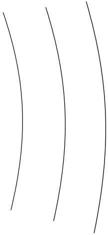

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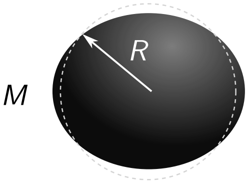

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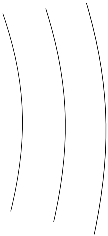

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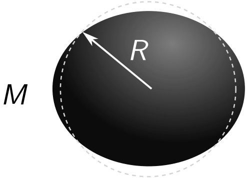

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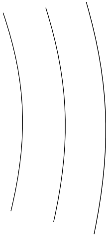

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$$U = \frac{M}{r} - \sum_{\ell \geqslant 2} \frac{(\ell-2)!}{\ell!} x^{a_1} \cdots x^{a_\ell} \mathcal{E}_{a_1 \cdots a_\ell} \left[1 + 2 k_\ell \left(\frac{R}{r} \right)^{2\ell+1} \right]$$

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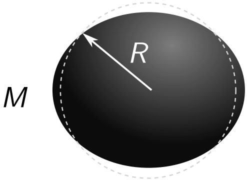

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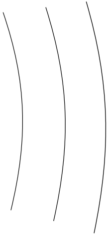

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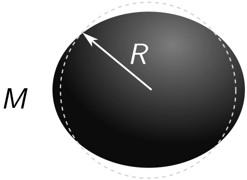

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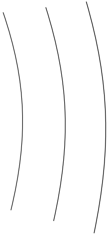

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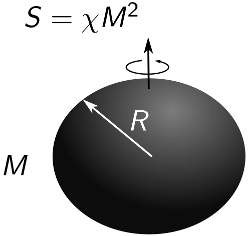

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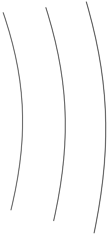

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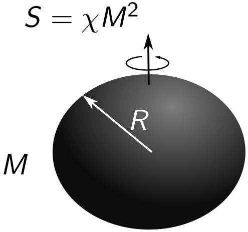

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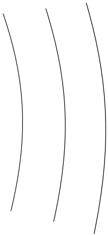

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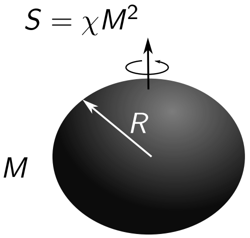
$$k_{\ell m} = k_\ell^{(0)} + i m \chi\, k_\ell^{(1)} + O(\chi^2)$$

Newtonian theory of Love numbers



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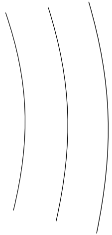

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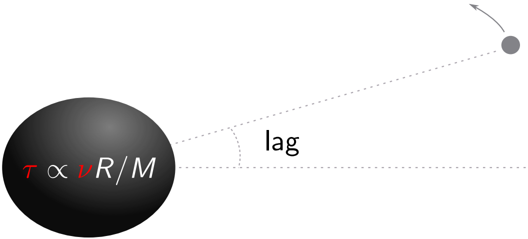
Tidal Love numbers $k_{\ell m} \longleftrightarrow$ **body's internal structure**

Viscosity and tidal dissipation



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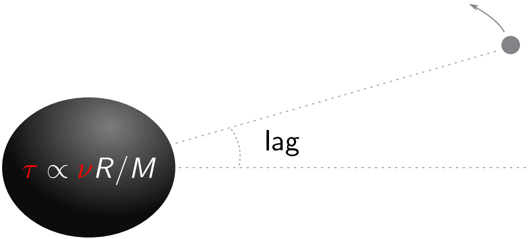

- Tidal lag:

$$Q_{ab}(t) = -\frac{2}{3}k_2 R^5 \left[\mathcal{E}_{ab}(t) - \tau\dot{\mathcal{E}}_{ab}(t) + \cdots\right]$$
$$= -\frac{2}{3}k_2 R^5 \left[\mathcal{E}_{ab}(t-\tau) + \cdots\right]$$

- Tidal torquing:

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc}\langle Q_{bd}\mathcal{E}^d{}_c\rangle = \frac{2}{3}(k_2\tau)R^5\,\varepsilon^{abc}\langle \dot{\mathcal{E}}_{bd}\mathcal{E}^d{}_c\rangle$$

Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_{a_1\cdots a_\ell} \propto [C_{0a_10a_2;a_3\cdots a_\ell}]^{\mathrm{STF}}\,, \quad \mathcal{B}_{a_1\cdots a_\ell} \propto [\varepsilon_{a_1bc}C_{a_20bc;a_3\cdots a_\ell}]^{\mathrm{STF}}$$

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- Metric and Geroch-Hansen multipole moments:

$$g_{\alpha\beta} = \mathring{g}_{\alpha\beta} + \underbrace{h^{\mathrm{tidal}}_{\alpha\beta}}_{\sim r^\ell} + \underbrace{h^{\mathrm{resp}}_{\alpha\beta}}_{\sim r^{-(\ell+1)}} \quad \longrightarrow \quad \begin{cases} M_{\ell m} = \mathring{M}_{\ell m} + \delta M_{\ell m} \\ S_{\ell m} = \mathring{S}_{\ell m} + \delta S_{\ell m} \end{cases}$$

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- *Four* families of tidal deformability parameters:

$$\lambda^{M\mathcal{E}}_{\ell\ell' m} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell' m}} \qquad \lambda^{S\mathcal{B}}_{\ell\ell' m} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell' m}}$$

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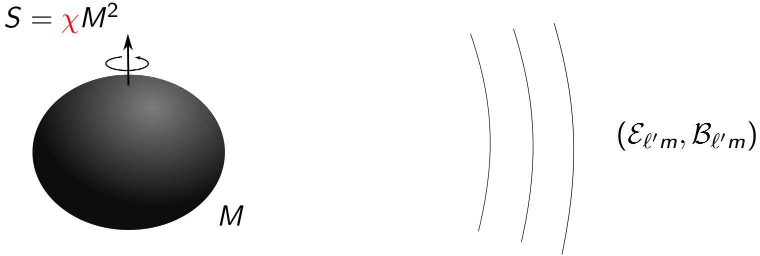
Investigating Kerr's Love



- Metric reconstruction through the Hertz potential Ψ:

$$(\mathcal{E}_{\ell' m}, \mathcal{B}_{\ell' m}) \to \psi_0 \to \Psi \to h_{\alpha\beta} \to (M_{\ell m}, S_{\ell m}) \to \lambda_{\ell\ell' m}^{M/S,\mathcal{E}/\mathcal{B}}$$

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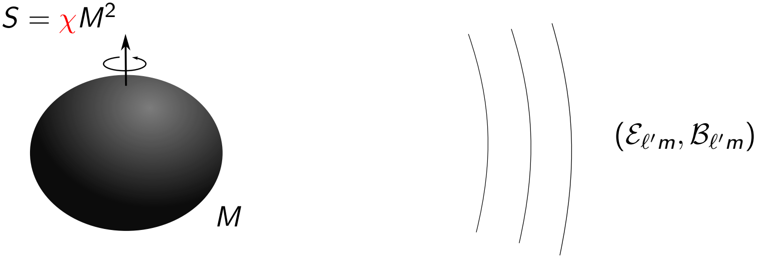

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- Quadrupolar tidal Love numbers of a Kerr black hole:

$$\lambda^{M\mathcal{E}}_{2\ell' m} = \lambda^{S\mathcal{B}}_{2\ell' m} \doteq \frac{im\chi}{180}(2M)^5\,\delta_{\ell' 2}\,, \quad \lambda^{M\mathcal{B}}_{2\ell' m} = \lambda^{S\mathcal{E}}_{2\ell' m} = 0$$

Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\rm el}\mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\rm mag}\mathcal{B}_{ab}$$

- For a spinning black hole we have the more general tensorial relations

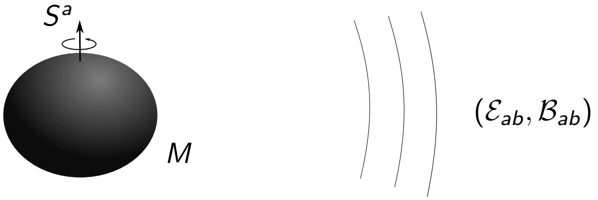
$$\delta M_{ab} = \lambda_{abcd}\mathcal{E}_{cd} \quad \text{and} \quad \delta S_{ab} = \lambda_{abcd}\mathcal{B}_{cd}$$

- To linear order in the black hole spin vector S^a we find

$$\delta M_{ab} \doteq \frac{16}{45}\, M^3\, S^c \mathcal{E}^d{}_{(a}\varepsilon_{b)cd}$$

$$\delta S_{ab} \doteq \frac{16}{45}\, M^3\, S^c \mathcal{B}^d{}_{(a}\varepsilon_{b)cd}$$

Tidal torquing of a spinning black hole



- Any spinning body interacting with a tidal environment suffers a tidal torquing [Thorne & Hartle 1980]

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc} \langle M_{bd} \mathcal{E}^d{}_c + S_{bd} \mathcal{B}^d{}_c \rangle$$

- Applied to a spinning black hole this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi \left[2 \langle \mathcal{E}^{ab} \mathcal{E}_{ab} \rangle - 3 \langle \mathcal{E}_{ab} s^b \mathcal{E}^{ac} s_c \rangle + (\mathcal{E} \to \mathcal{B}) \right]$$

- Full agreement with independant calculation by [Poisson 2004]

Tidal deformability and horizon viscosity

- A black hole has surface shear and bulk viscosity [Damour 1982]

$$\eta_S = \frac{1}{16\pi} \quad \text{and} \quad \zeta_S = -\frac{1}{16\pi}$$

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- Revisit analogy with nonzero Kerr black hole tidal Love numbers and horizon surface viscosity

Summary

- Love numbers of Kerr black holes do not vanish in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- Kerr black holes deform like any other self-gravitating body, despite being particularly "rigid" compact objects
- This is closely related to the phenomenon of tidal torquing
- Tidal deformability $\leftrightarrow$ horizon viscosity $\leftrightarrow$ tidal torquing

Spinning black holes fall in Love!

Black holes have *zero* Love numbers

Reference	**Background**	**Tidal field**
[Binnington & Poisson 2009]	Schwarzschild	weak, generic ℓ
[Damour & Nagar 2009]	Schwarzschild	weak, generic ℓ
[Kol & Smolkin 2012]	Schwarzschild	weak, electric-type
[Chakrabarti et al. 2013]	Schwarzschild	weak, electric, $\ell = 2$
[Gürlebeck 2015]	Schwarzschild	strong, axisymmetric
[Landry & Poisson 2015]	Kerr to $O(S)$	weak, quadrupolar
[Pani et al. 2015]	Kerr to $O(S^2)$	weak, $(\ell, m) = (2, 0)$
[Chia 2020]	Exact Kerr	weak, generic ℓ
[Goldberger & Rothstein 2020]	Exact Kerr	weak, generic ℓ
[Charalambous et al. 2021]	Exact Kerr	weak, generic ℓ

Perturbed Weyl scalar

- Recall that in the Newtonian limit we established

$$\lim_{c\to\infty} \psi_0^{\ell m} \propto \mathcal{E}_{\ell m}\, r^{\ell-2} \left[1 + 2k_{\ell m} \left(R/r\right)^{2\ell+1}\right] {}_2Y_{\ell m}(\theta,\phi)$$

- For a Kerr black hole the perturbed Weyl scalar reads

$$\psi_0^{\ell m} \propto \left[\mathcal{E}_{\ell m} + \tfrac{3i}{\ell+1}\, \mathcal{B}_{\ell m}\right] R_{\ell m}(r)\, {}_2Y_{\ell m}(\theta,\phi)$$

- Asymptotic behavior of general solution of static radial Teukolsky equation:

$$R_{\ell m}(r) = \underbrace{r^{\ell-2}\left(1+\cdots\right)}_{\text{tidal field } R^{\text{tidal}}_{\ell m}} + \kappa_{\ell m}\, \underbrace{r^{-\ell-3}\left(1+\cdots\right)}_{\text{linear response } R^{\text{resp}}_{\ell m}}$$

Kerr black hole linear response

$$R_{\ell m}(r) = \underbrace{R^{\text{tidal}}_{\ell m}(r)}_{\sim r^{\ell-2}} + 2k_{\ell m}\underbrace{R^{\text{resp}}_{\ell m}(r)}_{\sim r^{-(\ell+3)}}$$

- The coefficients $k_{\ell m}$ can be interpreted as the Newtonian Love numbers of a Kerr black hole and read

$$k_{\ell m} = -im\chi\,\frac{(\ell+2)!(\ell-2)!}{4(2\ell+1)!(2\ell)!}\prod_{n=1}^{\ell}\left[n^2(1-\chi^2)+m^2\chi^2\right]$$

- The linear response vanishes identically when:
  - the black hole spin vanishes ($\chi = 0$)
  - the tidal field is axisymmetric ($m = 0$)
- Reconstruct the Kerr black hole response $h^{\text{resp}}_{\alpha\beta}$ via Ψ^{resp}

Love numbers of a Kerr black hole

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- The black hole tidal bulge is rotated by 45° with respect to the quadrupolar tidal perturbation
- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} \doteq 0$$

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- We compute the Love numbers to linear order in $\chi \equiv S/M^2$
- The modes of the mass/current quadrupole moments are

$$\delta M_{2m} \doteq \frac{im\chi}{180}(2M)^5\,\mathcal{E}_{2m} \quad \text{and} \quad \delta S_{2m} \doteq \frac{im\chi}{180}(2M)^5\,\mathcal{B}_{2m}$$

- The black hole tidal bulge is rotated by 45° with respect to the quadrupolar tidal perturbation
- The associated dimensionless tidal Love numbers are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} \doteq -\frac{im\chi}{120} \quad \text{and} \quad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} \doteq 0$$

- For a dimensionless black hole spin $\chi = 0.1$ this gives

$$|k_{2,\pm 2}| \simeq 2\times 10^{-3} \quad \longrightarrow \quad \text{black holes are } \textbf{“rigid”}$$

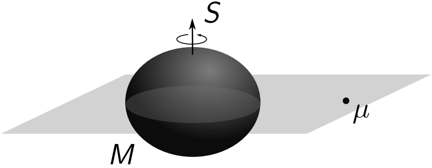
Love tensor of a Kerr black hole

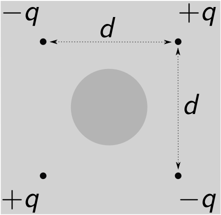
$$(\lambda_{abcd}) \doteq \frac{\chi}{180}\,(2M)^5 \begin{pmatrix} \mathbf{I}_{11} & \mathbf{I}_{12} & \mathbf{I}_{13} \\ \mathbf{I}_{12} & -\mathbf{I}_{11} & \mathbf{I}_{23} \\ \mathbf{I}_{13} & \mathbf{I}_{23} & \mathbf{0} \end{pmatrix}$$

$$\mathbf{I}_{11} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{I}_{12} \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{I}_{13} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \mathbf{I}_{23} \equiv \begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

Newtonian static quadrupolar tide





$$\mathcal{E}_{ab} = \frac{\mu}{r^3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\delta M_{ab} \doteq 3Q \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\uparrow$

$$\tfrac{\chi}{180}\,(2M)^5\,\tfrac{\mu}{r^3} = qd^2$$