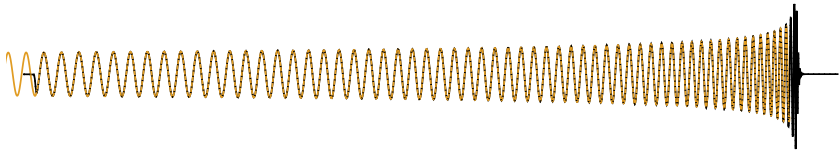


# Gravitational waveforms for intermediate mass ratio inspirals

Alexandre Le Tiec

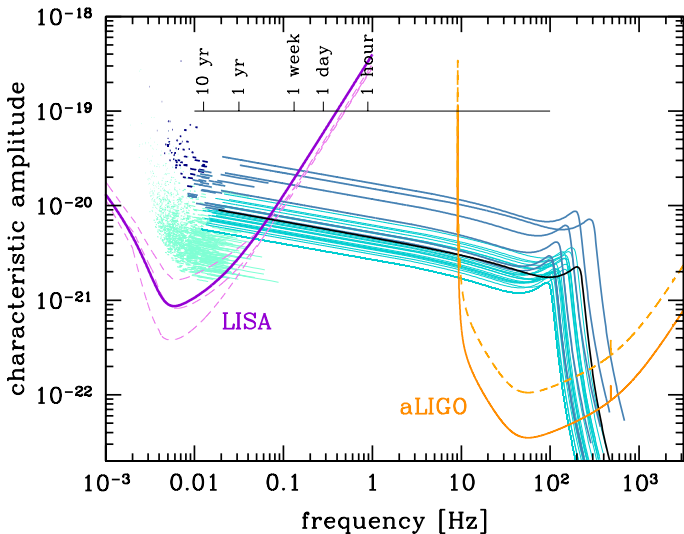
Laboratoire Univers et Théories  
Observatoire de Paris / CNRS

Collaborators: L. Durkan, J. Miller,  
A. Pound, N. Warburton, B. Wardell



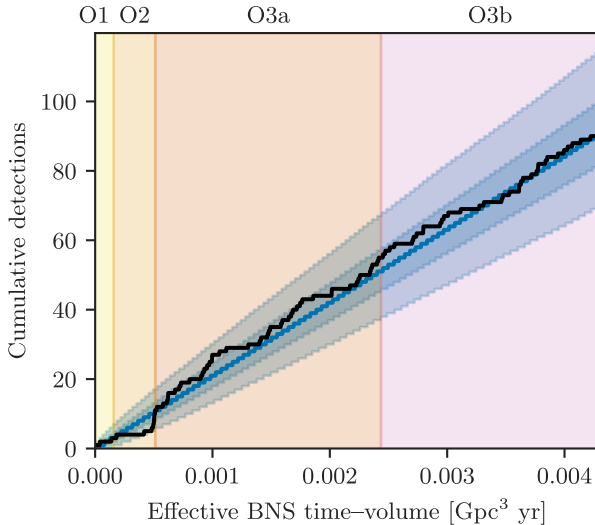
# Multi-band gravitational-wave astronomy

[Sesana PRL 2016]



# Current gravitational-wave detections

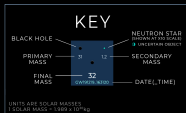
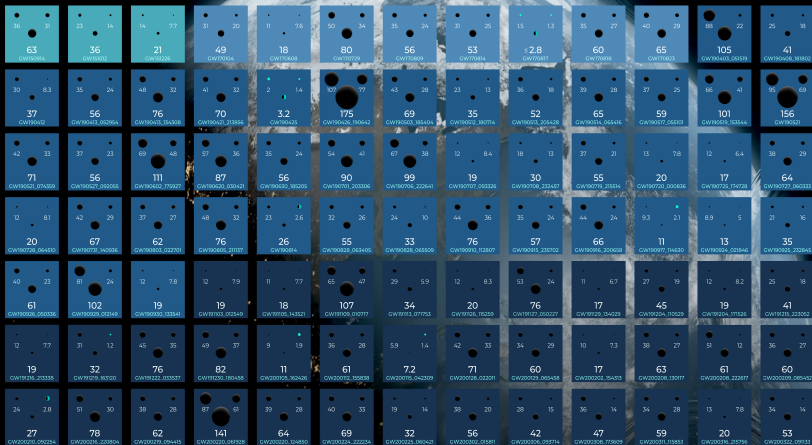
[LIGO-Virgo-KAGRA 2021]



OBSERVING  
01  
2015 - 2016

02  
2016 - 2017

03a+b  
2019 - 2020



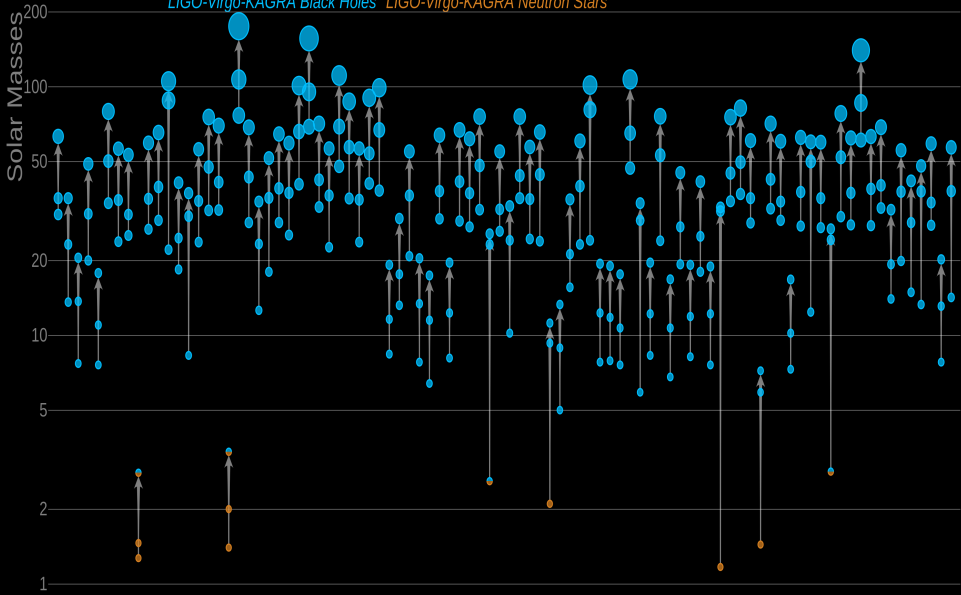
# GRAVITATIONAL WAVE MERGER DETECTIONS

SINCE 2015



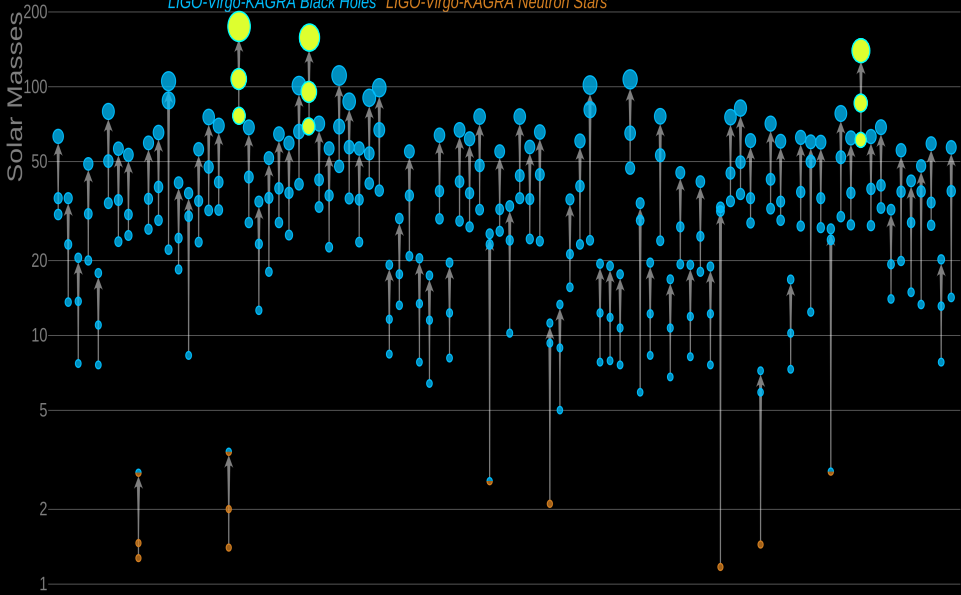
# Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars



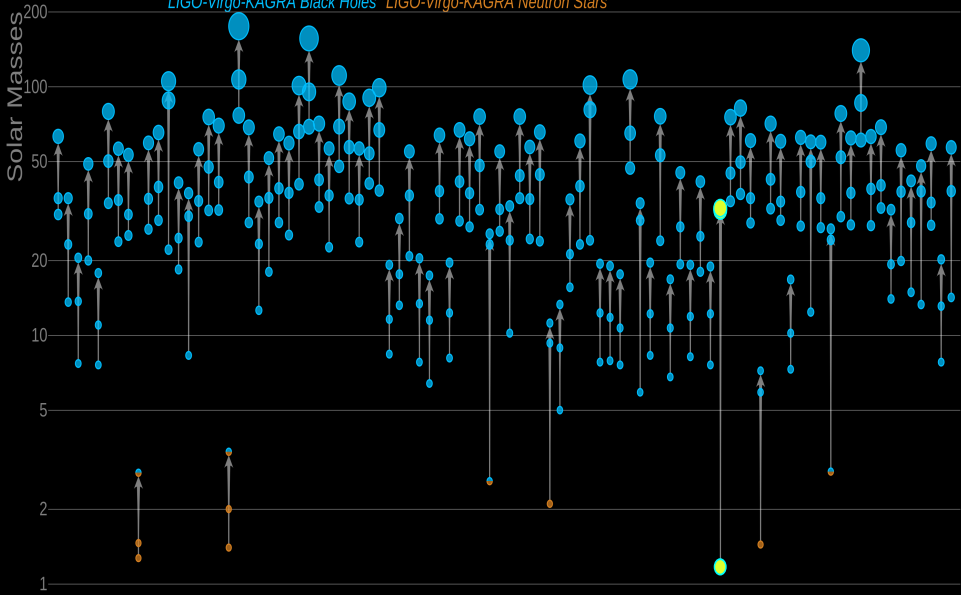
# Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars



# Masses in the Stellar Graveyard

*LIGO-Virgo-KAGRA Black Holes* *LIGO-Virgo-KAGRA Neutron Stars*



# Systematic uncertainties in modeling IMRIs

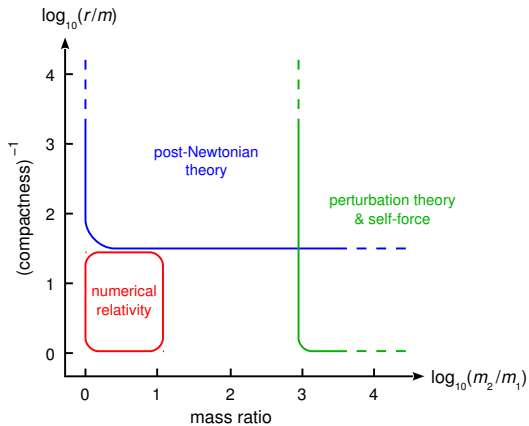
[LIGO-Virgo-KAGRA 2021]

*The mass ratio of GW191219\_163120's source is inferred to be  $q = 0.038^{+0.005}_{-0.004}$ , which is **extremely challenging** for waveform modeling, and thus there may be **systematic uncertainties** in results for this candidate.*

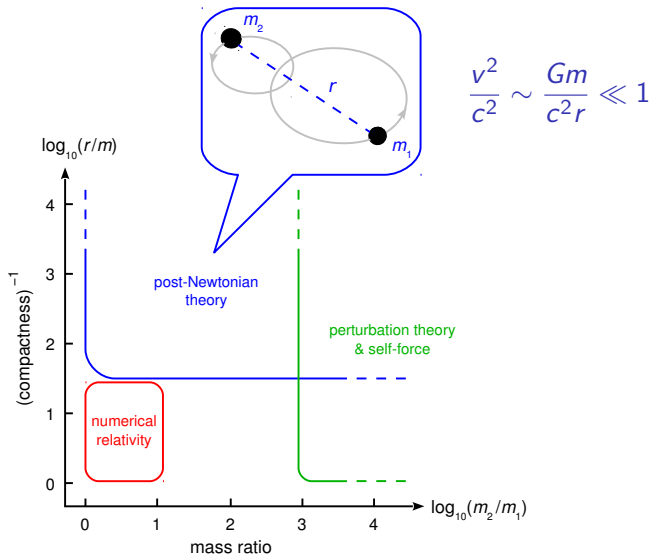
*Modeling of **higher-order multipole moments** is particularly important for inferring the properties of systems with unequal masses, and may **impact inference of parameters** including the mass ratio, inclination and distance.*



# Modeling coalescing compact binaries

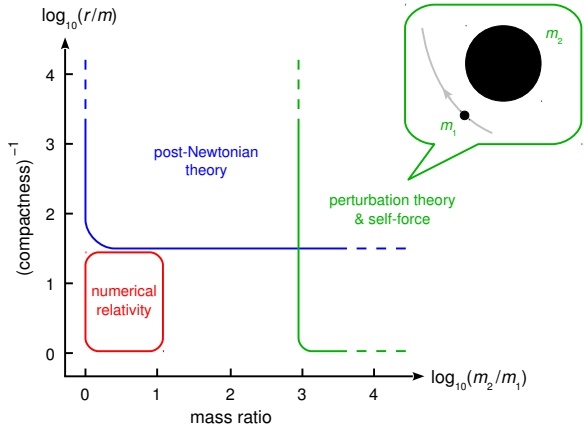


# Modeling coalescing compact binaries

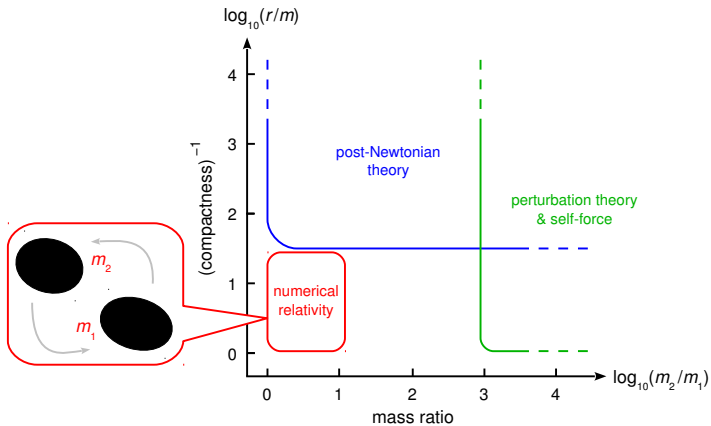


# Modeling coalescing compact binaries

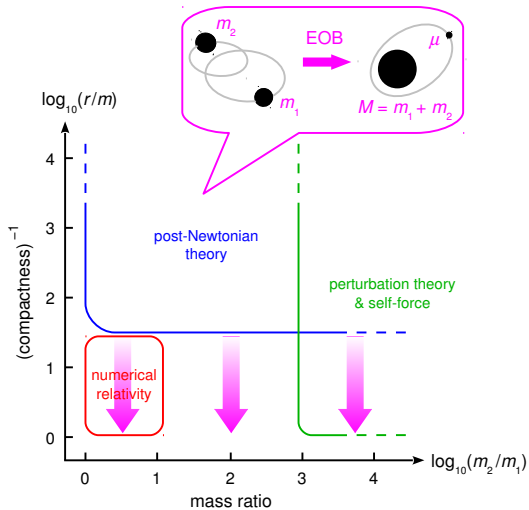
$$q \equiv \frac{m_1}{m_2} \ll 1$$



# Modeling coalescing compact binaries

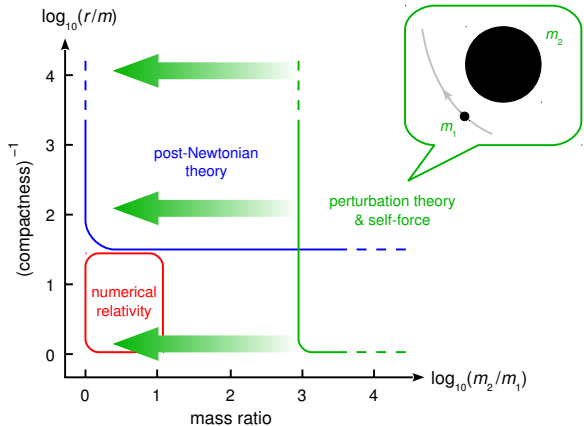


# Modeling coalescing compact binaries



# Modeling coalescing compact binaries

$$q \equiv \frac{m_1}{m_2} \rightarrow \nu \equiv \frac{m_1 m_2}{m^2}$$



# Perturbation theory for comparable masses

## Relativistic orbital dynamics

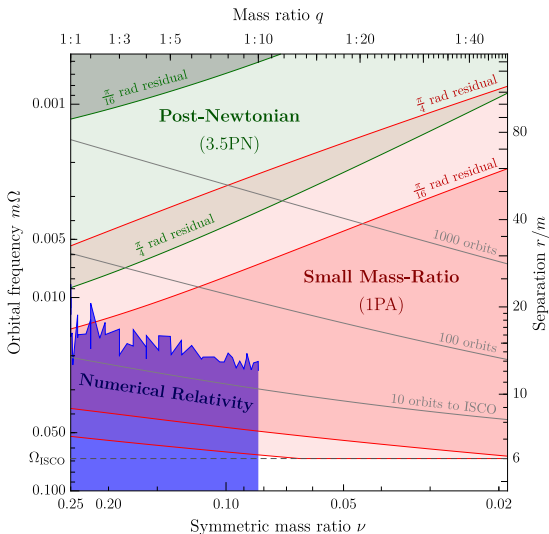
- Periastron advance [Le Tiec *et al.* PRL 2011; PRD 2013]
- Binding energy [Le Tiec, Buonanno & Barausse PRL 2012]
- Surface gravity [Le Tiec & Grandclément CQG 2018]

## Gravitational-wave emission

- Recoil velocity [Fitchett & Detweiler ApJ 1984, Nagar PRD 2013]
- Head-on waveform [Anninos *et al.* PRD 1995, Sperhake *et al.* PRD 2011]
- Inspiral phasing [van de Meent & Pfeiffer PRL 2020]
- Inspiral energy flux [Warburton *et al.* PRL 2021]

# Perturbation theory for comparable masses

[van de Meent & Pfeiffer PRL 2020]





## Second order gravitational self-force program

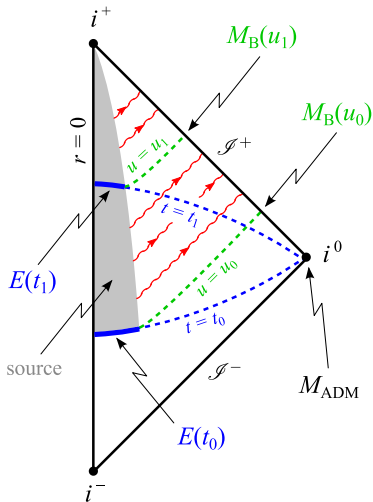
- *Second-order gravitational self-force*  
A. Pound, PRL **109** (2012) 051101
- *Practical, covariant puncture for second-order self-force calculations*  
A. Pound & J. Miller, PRD **89** (2014) 104020
- *Second-order perturbation theory: Problems on large scales*  
A. Pound, PRD **92** (2015) 104047
- *Second-order perturbation theory: The problem of infinite mode coupling*  
J. Miller, B. Wardell & A. Pound, PRD **94** (2016) 104018
- *Nonlinear gravitational self-force: Second-order equation of motion*  
A. Pound, PRD **95** (2017) 104056
- *Second-order self-force calculation of gravitational binding energy in compact binaries*  
A. Pound, B. Wardell, N. Warburton & J. Miller, PRL **124** (2019) 021101
- *Two-timescale evolution of extreme-mass-ratio inspirals: Waveform generation scheme for quasicircular orbits in Schwarzschild spacetime*  
J. Miller & A. Pound, PRD **103** (2021) 064048
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N. Warburton, A. Pound, B. Wardell et al., PRL **127** (2021) 151102

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# Orbital evolution *via* energy balance

[Bondi *et al.* 1962; Sachs 1962]



- Bondi mass-loss formula

$$\frac{dM_B}{du} = -\mathcal{F}(u)$$

- Gravitational binding energy

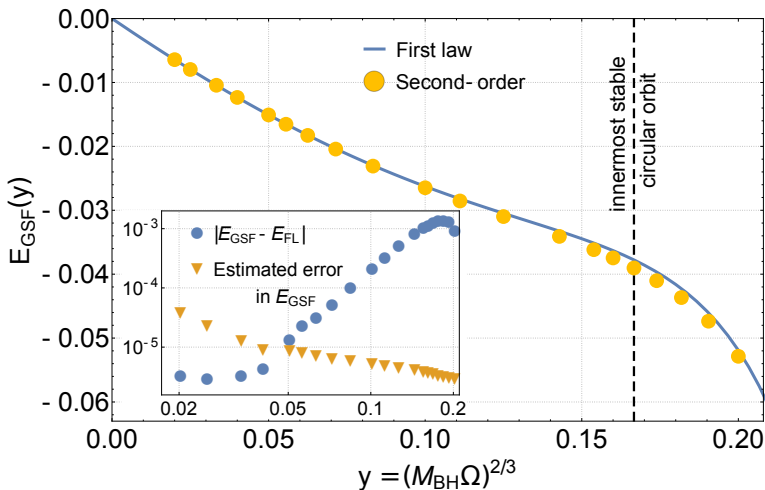
$$E \equiv M_B - M_{\text{BH}} - \mu$$

- Orbital frequency evolution

$$\frac{d\omega}{dt} = -\frac{\mathcal{F}(\omega)}{E'(\omega)}$$

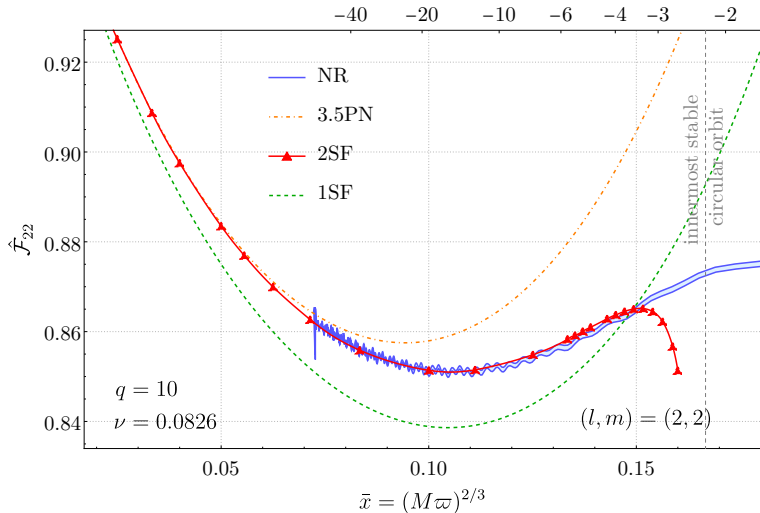
# Second-order binding energy

[Pound *et al.* PRL 2020]



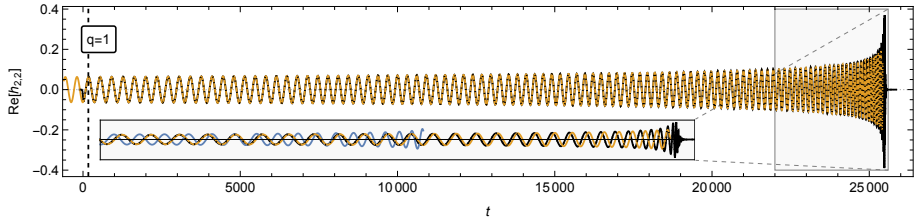
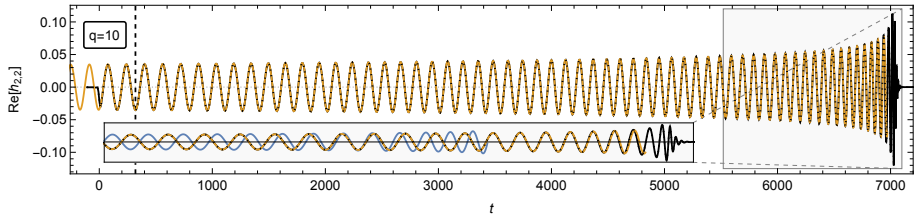
# Second-order energy flux

[Warburton *et al.* PRL 2021]



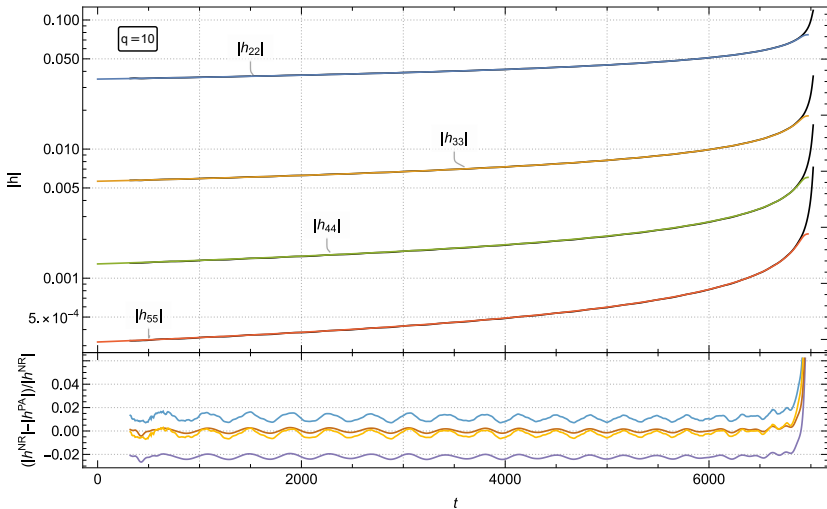
# Gravitational waveforms

[Wardell *et al.* 2021]



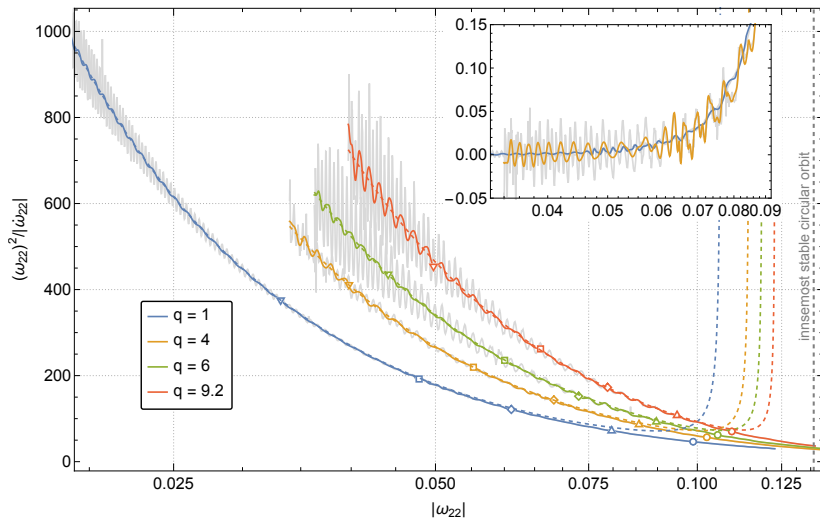
# Mode waveform amplitudes

[Wardell *et al.* 2021]



# Waveforms frequency evolution

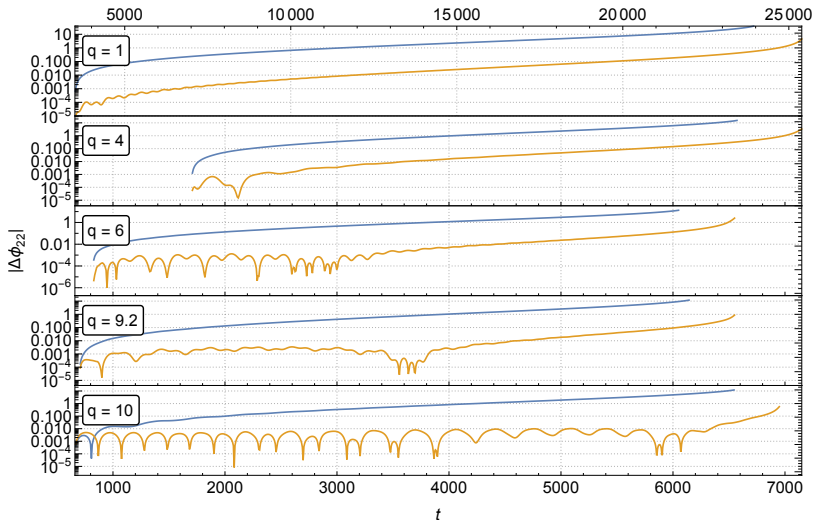
[Wardell *et al.* 2021]





# Accumulated dephasing

[Wardell *et al.* 2021]



## Summary

- **Intermediate** mass ratio inspirals (IMRIs) are promising gravitational-wave sources for LIGO-Virgo and **LISA**
- IMRIs are **challenging** for existing modeling techniques and current templates are **not reliable** for  $q \gtrsim 30$
- **Post-adiabatic waveforms** agree remarkably well with the results from full numerical relativity with  $1 \leq q \leq 10$
- Second-order gravitational **self-force theory** will be used to model EMRIs, IMRIs and possibly **comparable-mass** systems
- In the future: transition to **plunge** / add black hole **spin**