Spinning black holes fall in Love

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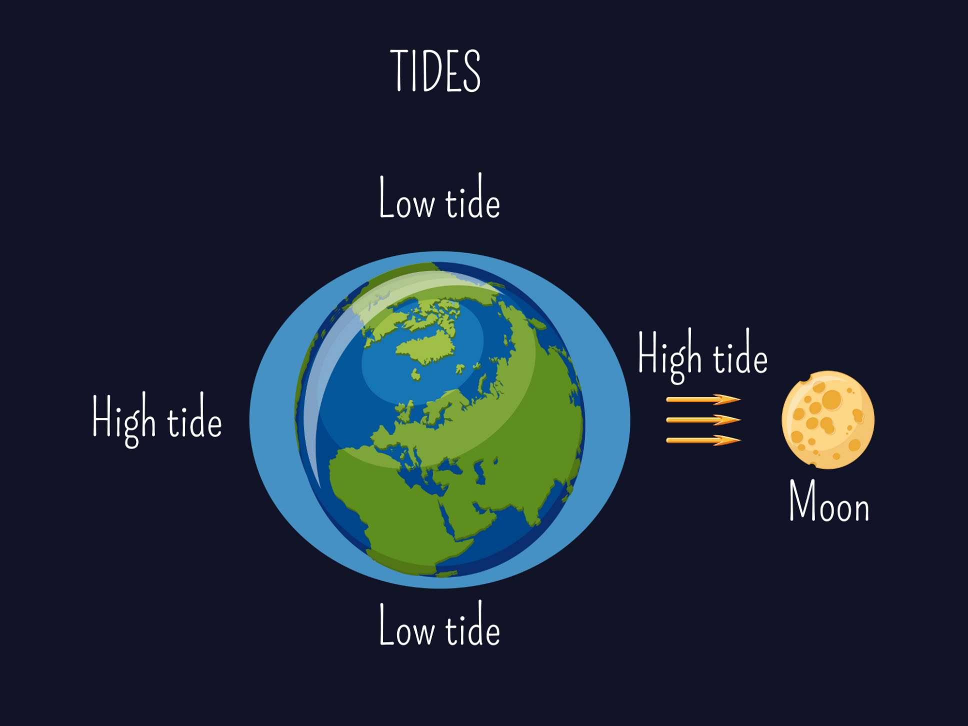
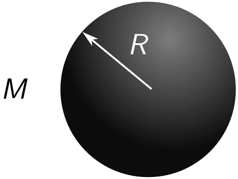
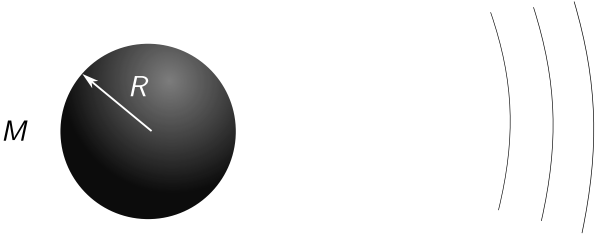

TIDES
Low tide
High tide
High tide
Moon
Low tide

Newtonian theory of Love numbers



$$U = \frac{M}{r}$$

Newtonian theory of Love numbers



$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(0)$$

$$U = \frac{M}{r} - \frac{1}{2} x^a x^b \mathcal{E}_{ab}$$

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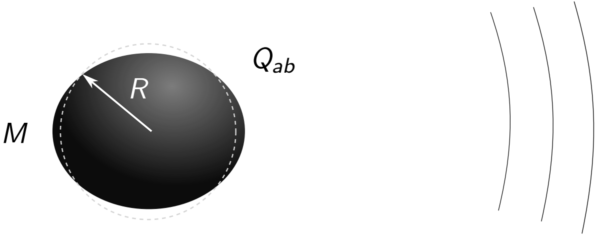

$$\mathcal{E}_{ab} = -\partial_a \partial_b U_{\text{ext}}(0)$$

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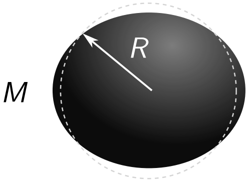

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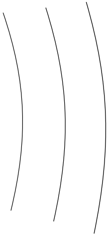

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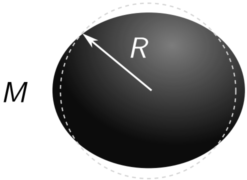

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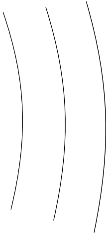

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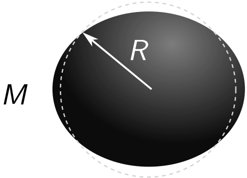

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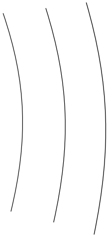

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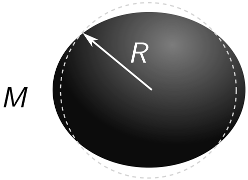
Newtonian theory of Love numbers



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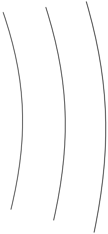

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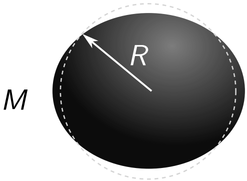

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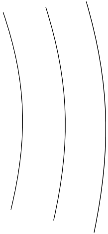

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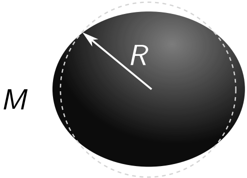

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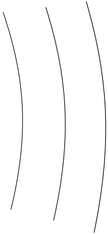

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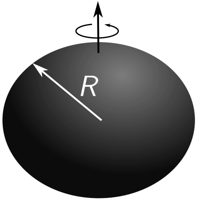
Newtonian theory of Love numbers

$S = \chi M^2$

M



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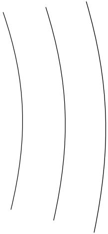

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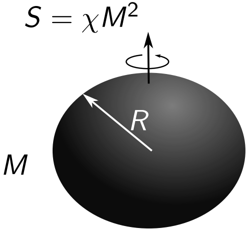

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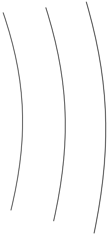

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$$k_{\ell m} = k_\ell^{(0)} + i m \chi\, k_\ell^{(1)} + O(\chi^2)$$

Newtonian theory of Love numbers

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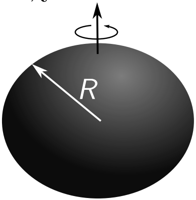

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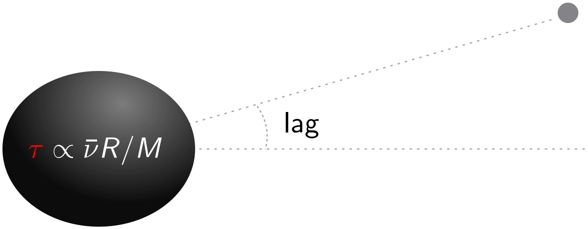
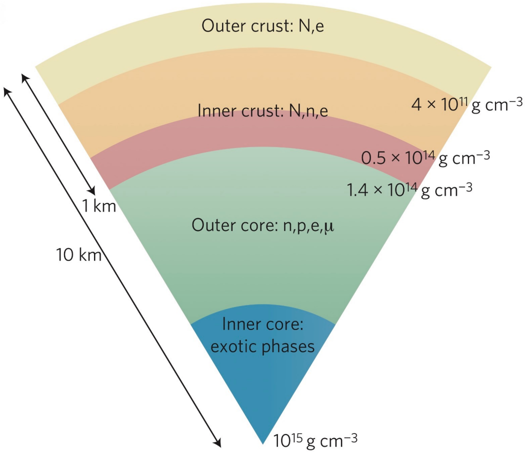
Tidal Love numbers $k_{\ell m} \longleftrightarrow$ **body's internal structure**

Tidal dissipation: lag, heating and torquing



$$Q_{ab}(t) = -\frac{2}{3} k_2 R^5 \left[\mathcal{E}_{ab}(t) - \tau \dot{\mathcal{E}}_{ab}(t) + \cdots \right]$$
$$= -\frac{2}{3} k_2 R^5 \left[\mathcal{E}_{ab}(t - \tau) + \cdots \right]$$

Internal structure of neutron stars

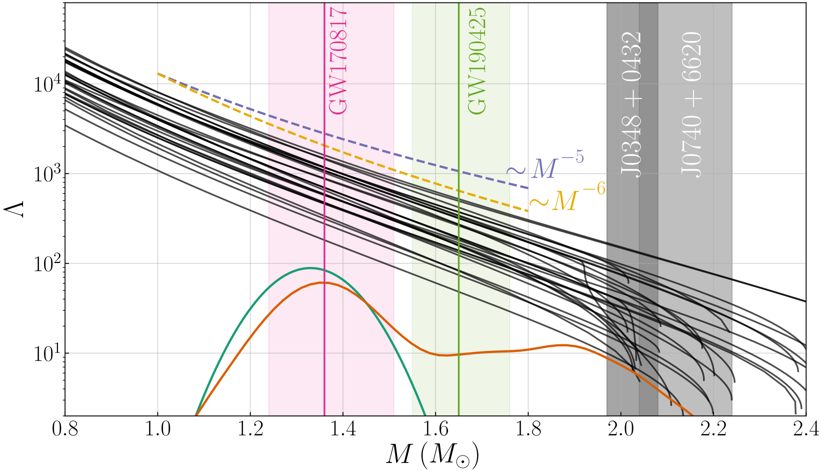


GW observations as probes of **neutron star internal structure**

Gravitational-wave observations

[Chatziioannou, GRG 2020]

$\Lambda \equiv \lambda_2/M^5 \propto k_2/C^5$



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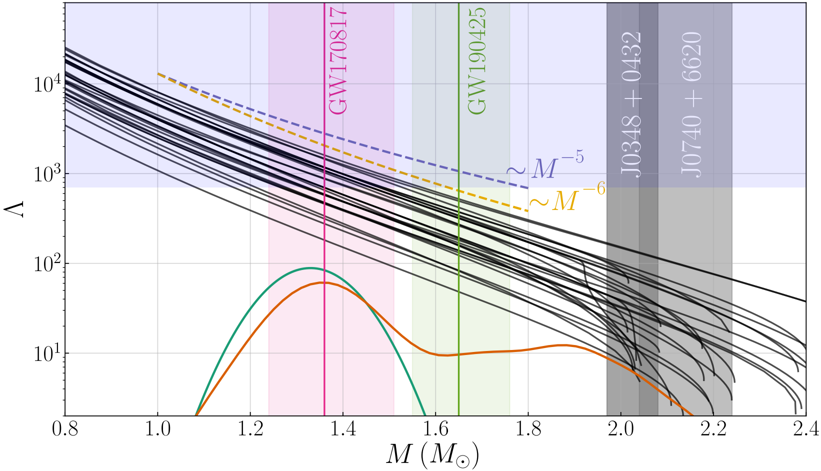

Relativistic theory of Love numbers

- Electric-type and magnetic-type tidal moments:

$$\mathcal{E}_L \propto [C_{0a_1 0a_2;a_3\cdots a_\ell}]^{\mathsf{STF}} \quad \text{and} \quad \mathcal{B}_L \propto [\varepsilon_{a_1 bc} C_{a_2 0bc;a_3\cdots a_\ell}]^{\mathsf{STF}}$$

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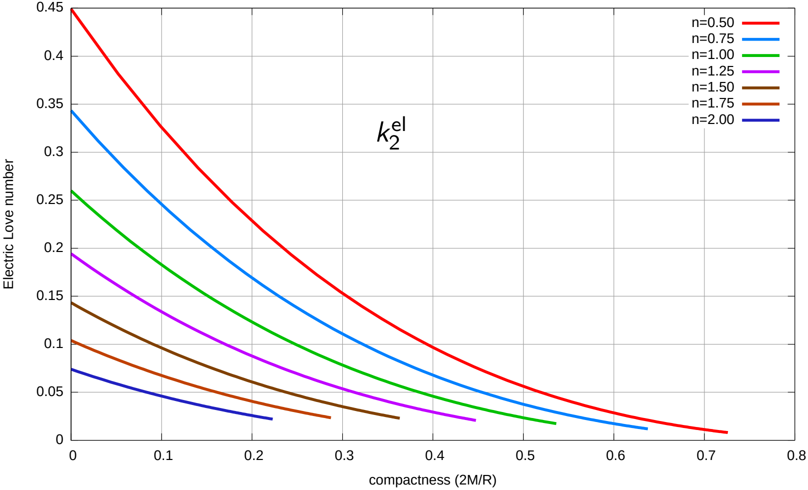
- Dimensionless tidal Love numbers:

$$k^{\text{el/mag}}_\ell \equiv -\frac{(2\ell-1)!!}{2(\ell-2)!}\,\frac{\lambda^{\text{el/mag}}_\ell}{R^{2\ell+1}}$$

Love numbers of neutron stars

[Binnington & Poisson, PRD 2009]



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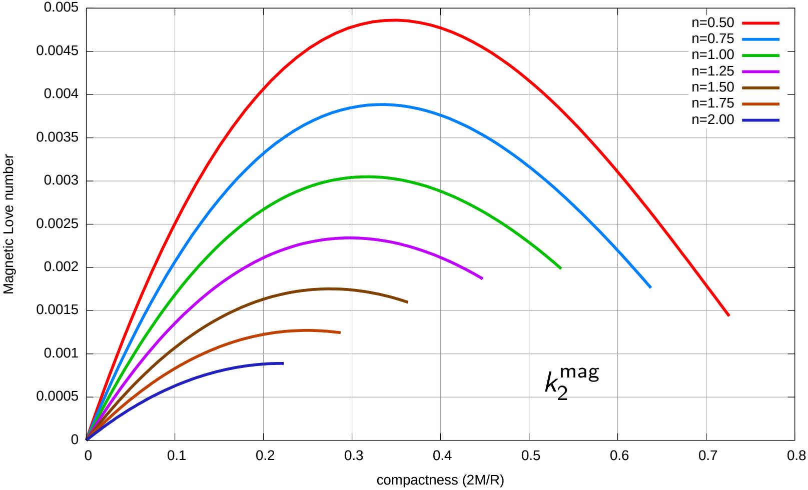

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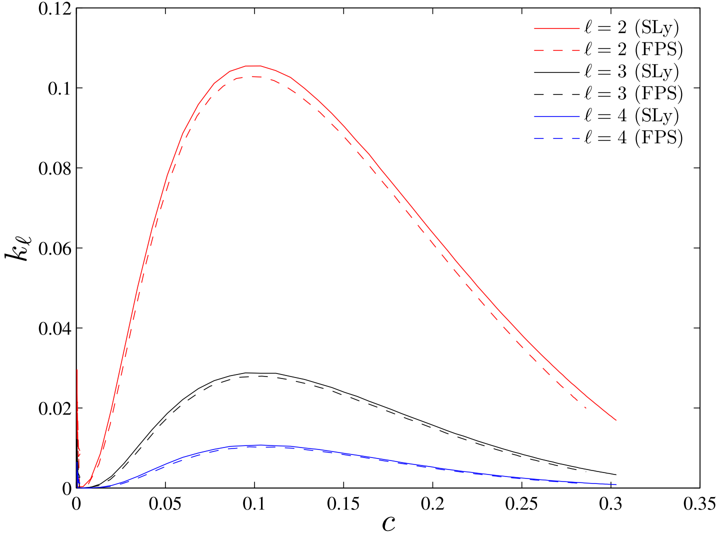

Love numbers of *spinning* compact objects

- The spin breaks the spherical symmetry of the background
  - No proportionality between $(\delta M_L, \delta S_L)$ and $(\mathcal{E}_L, \mathcal{B}_L)$
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- *Four* families of tidal deformability parameters:

$$\lambda^{M\mathcal{E}}_{\ell\ell' mm'} \equiv \frac{\partial \delta M_{\ell m}}{\partial \mathcal{E}_{\ell' m'}} \qquad \lambda^{S\mathcal{B}}_{\ell\ell' mm'} \equiv \frac{\partial \delta S_{\ell m}}{\partial \mathcal{B}_{\ell' m'}}$$

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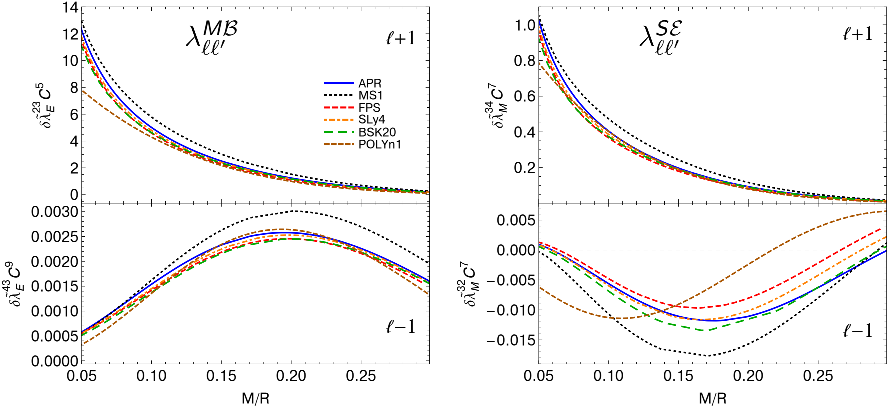
Love numbers of *spinning* neutron stars

[Pani, Gualtieri & Ferrari, PRD 2015]



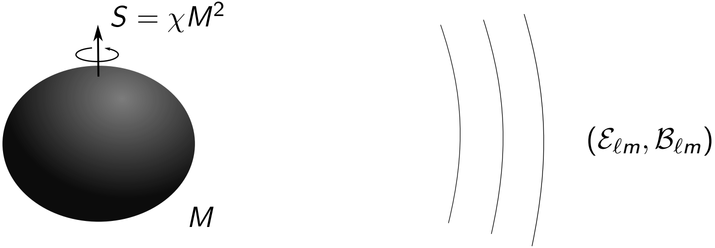
Restricted to an *axisymmetric* tidal perturbation ($m = 0$)

Black holes have *zero* Love numbers

Reference	**Background**	**Tidal field**
[Binnington & Poisson 2009]	Schwarzschild	weak, generic ℓ
[Damour & Nagar 2009]	Schwarzschild	weak, generic ℓ
[Kol & Smolkin 2012]	Schwarzschild	weak, electric-type
[Chakrabarti et al. 2013]	Schwarzschild	weak, electric, $\ell = 2$
[Gürlebeck 2015]	Schwarzschild	strong, axisymmetric
[Landry & Poisson 2015]	Kerr to $O(S)$	weak, quadrupolar
[Pani et al. 2015]	Kerr to $O(S^2)$	weak, $(\ell, m) = (2, 0)$
[Chia 2020]	Exact Kerr	weak, generic ℓ
[Goldberger & Rothstein 2020]	Exact Kerr	weak, generic ℓ
[Charalambous et al. 2021]	Exact Kerr	weak, generic ℓ

Problem of **fine-tuning** from an Effective-Field-Theory perspective

Investigating Kerr's Love



$$(\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}) \to \psi_0 \to \Psi \to h_{\alpha\beta} \to (M_{\ell m}, S_{\ell m}) \to \lambda_{\ell m}^{M/S,\mathcal{E}/\mathcal{B}}$$

Metric reconstruction through the Hertz potential Ψ

Perturbed Weyl scalar

- Recall that in the Newtonian limit we established

$$\lim_{c\to\infty} \psi_0^{\ell m} \propto \mathcal{E}_{\ell m}\, r^{\ell-2} \left[1 + 2k_{\ell m}\,(R/r)^{2\ell+1}\right] {}_2Y_{\ell m}(\theta,\phi)$$

- For a Kerr black hole the perturbed Weyl scalar reads

$$\psi_0^{\ell m} \propto \left[\mathcal{E}_{\ell m} + \tfrac{3i}{\ell+1}\,\mathcal{B}_{\ell m}\right] R_{\ell m}(r)\; {}_2Y_{\ell m}(\theta,\phi)$$

- Asymptotic behavior of general solution of static radial Teukolsky equation:

$$R_{\ell m}(r) = \underbrace{r^{\ell-2}\,(1+\cdots)}_{\text{tidal field } R^{\text{tidal}}_{\ell m}} + k_{\ell m}\,\underbrace{r^{-\ell-3}\,(1+\cdots)}_{\text{linear response } R^{\text{resp}}_{\ell m}}$$

Why analytic continuation?

$$R_{\ell m}(r) = \underbrace{r^{\ell-2}\,(1+\cdots)}_{\text{tidal field } R_{\ell m}^{\text{tidal}}} + k_{\ell m}\,\underbrace{r^{-\ell-3}\,(1+\cdots)}_{\text{linear response } R_{\ell m}^{\text{resp}}}$$

Ambiguity in the linear response [Fang & Lovelace 2005; Gralla 2018]

The decaying solution $R_{\ell m}^{\text{resp}}$ is affected by a radial coord. transfo.

Ambiguity in the tidal field [Pani, Gualtieri, Maselli & Ferrari 2015]

The growing solution $R_{\ell m}^{\text{tidal}} + \alpha R_{\ell m}^{\text{resp}}$ still qualifies as a tidal solution

Kerr black hole linear response

$$R_{\ell m}(r) = \underbrace{R_{\ell m}^{\text{tidal}}(r)}_{\sim r^{\ell-2}} + 2k_{\ell m}\underbrace{R_{\ell m}^{\text{resp}}(r)}_{\sim r^{-(\ell+3)}}$$

- The coefficients $k_{\ell m}$ can be interpreted as the Newtonian Love numbers of a Kerr black hole and read

$$k_{\ell m} = -im\chi\,\frac{(\ell+2)!(\ell-2)!}{4(2\ell+1)!(2\ell)!}\prod_{n=1}^{\ell}\left[n^2(1-\chi^2)+m^2\chi^2\right]$$

- The linear response vanishes identically when:
  - the black hole spin vanishes ($\chi = 0$)
  - the tidal field is axisymmetric ($m = 0$)
- Reconstruct the Kerr black hole response $h_{\alpha\beta}^{\text{resp}}$ via Ψ^{resp}

Love numbers of a Kerr black hole

- We compute the Love numbers to linear order in $\chi \equiv S/M^2$

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- The modes of the mass/current quadrupole moments are

$$\delta M_{2m} \doteq \frac{im\chi}{180}\,(2M)^5\,\mathcal{E}_{2m} \quad \text{and} \quad \delta S_{2m} \doteq \frac{im\chi}{180}\,(2M)^5\,\mathcal{B}_{2m}$$

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- For a dimensionless black hole spin $\chi = 0.1$ this gives

$$|k_{2,\pm 2}| \simeq 2 \times 10^{-3} \quad \longrightarrow \quad \text{black holes are } \textbf{“rigid”}$$

Love tensor of a Kerr black hole

- For a nonspinning compact body we have the proportionality relations

$$\delta M_{ab} = \lambda_2^{\mathrm{el}} \mathcal{E}_{ab} \quad \text{and} \quad \delta S_{ab} = \lambda_2^{\mathrm{mag}} \mathcal{B}_{ab}$$

- For a spinning black hole we have the more general tensorial relations

$$\delta M_{ab} = \lambda_{abcd} \mathcal{E}_{cd} \quad \text{and} \quad \delta S_{ab} = \lambda_{abcd} \mathcal{B}_{cd}$$

- To linear order in the black hole spin vector S^a we find

$$\delta M_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{E}^d{}_{(a} \varepsilon_{b)cd}$$

$$\delta S_{ab} \doteq \frac{16}{45} M^3 S^c \mathcal{B}^d{}_{(a} \varepsilon_{b)cd}$$

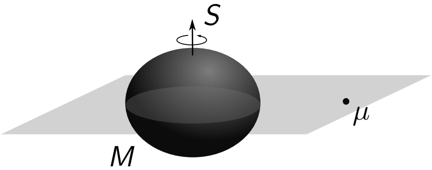
Love tensor of a Kerr black hole

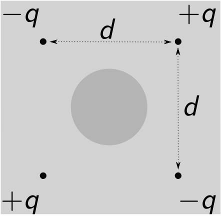
$$(\lambda_{abcd}) \doteq \frac{\chi}{180}\,(2M)^5 \begin{pmatrix} \mathsf{I}_{11} & \mathsf{I}_{12} & \mathsf{I}_{13} \\ \mathsf{I}_{12} & -\mathsf{I}_{11} & \mathsf{I}_{23} \\ \mathsf{I}_{13} & \mathsf{I}_{23} & 0 \end{pmatrix}$$

$$\mathsf{I}_{11} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathsf{I}_{12} \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathsf{I}_{13} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \quad \mathsf{I}_{23} \equiv \begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

Newtonian static quadrupolar tide





$$\mathcal{E}_{ab} = \frac{\mu}{r^3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

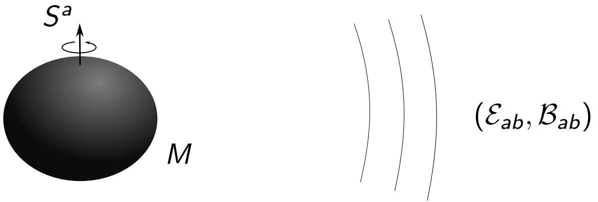
$$\delta M_{ab} \doteq 3Q \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\uparrow$$

$$\frac{\chi}{180} (2M)^5 \frac{\mu}{r^3} = qd^2$$

Tidal torquing of a spinning black hole

[Thorne & Hartle 1980; Poisson 2004]



- An arbitrary spinning body interacting with a tidal environment suffers a tidal torquing:

$$\langle \dot{S}^a \rangle = -\varepsilon^{abc} \langle M_{bd} \mathcal{E}^d{}_c + S_{bd} \mathcal{B}^d{}_c \rangle$$

- Applied to a spinning black hole this yields

$$\langle \dot{S} \rangle \doteq -\frac{8}{45} M^5 \chi \left[2\langle E_1 + B_1 \rangle - 3\langle E_2 + B_2 \rangle \right]$$

Summary

- Love numbers of Kerr black holes do not vanish in general
- We computed in closed-form the leading (quadrupolar) Love numbers to linear order in the black hole spin
- Kerr black holes deform like any other self-gravitating body, despite being particularly "rigid" compact objects
- This is closely related to the phenomenon of tidal torquing
- New black hole test of the Kerr-like nature of the massive compact objects at the center of galaxies?

Spinning black holes fall in Love!